VSB — TECHNICAL UNIVERSITY OF OSTRAVA FACULTY OF ECONOMICS

DEPARTMENT OF FINANCE

Využití Matlabu při Optimalizaci Portfolia

Application of Matlab in Portfolio Optimization

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1. Introduction
2. Description of Matlab
3. Description of Portfolio Optimization
4. Application of Portfolio Optimization in Matlab
5. Conclusion

Bibliography
List of Abbreviations
Declaration of Utilisation of Results from the Diploma Thesis
List of Annexes
Annexes

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The declaration

"Herewith I declare that I elaborated the entire thesis, including all annexes independently."

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Contents

1 Introduction

There exists a rule in investing activities, which is “don’t put all eggs in one basket”. It’s because of the existence of this rule, the portfolio optimization becomes important. As we all know, portfolio optimization is a process of selecting the best portfolio that meet the investor’s requirement. The main idea of the portfolio optimization is how investors make a choice between the risk and return. In general, the investors want to maximum the return at a low-level risk, but in fact, the rise in return is also followed by the increasement of risk. The portfolio optimization was firstly proposed by Markowitz (1952), who proposed that if the investor need to make a decision between two portfolios with the same return, all investors will choose the portfolio with less risky.

The goal of the thesis is to compare the performance for portfolios with different models in different sample periods. There are four models that will be used in the thesis: max Sharpe ratio model, mean-variance model, random model and naive strategy. For this study, we choose 30 stocks that are components of Dow Jones Industrial Average from the Yahoo website and apply the data from 2008 to 2018. What’s more, in order to complete the study well, we apply the Matlab as a main calculation tool.

The thesis can be divided into five chapters. As you see, the first chapter is introduction. The second chapter described the basic operation knowledge of Matlab, it includes the interface introduction, computing rules, programs debugging and graphing in Matlab.

The third chapter described the portfolio optimization model what we apply. Firstly, the characters of stock portfolio will be mentioned, such as mean return, standard deviation, skewness, kurtosis. Secondly, risk measure will be introduced, it includes standard deviation, value at risk and conditional value at risk. Thirdly, naive strategy, mean-variance model, random model and max Sharpe ratio will be explained carefully. The fourthly is wealth calculation. And the last one is performance measurement, these theories are used to evaluate the performance of portfolio.

The fourth chapter is application part. In this part, all models that we have mentioned in pervious chapter will be applied by using Matlab, and all the results of calculation will be shown in this chapter.

Finally, the last chapter is conclusion, we will make a summary according to our calculation for the whole thesis.

2. Description of Matlab

Matlab, a kind of mathematics software, was produced by the MathWorks company. As a high-level technical computing language, Matlab can be used for data analysis, data visualization and numerical computation. Besides the basic function of matrix operation, make function and draw the image, Matlab also can be used to create the user interface and write programs in other technical computing languages, such as C, C++， java and so on.

Although Matlab is mainly used for numerical computation, it takes many additional toolboxes, which are also suitable for applications in different fields. Each toolbox is a set of function that can implement a specific function. The toolbox provided by MathWorks is divided into some categories, such as the control system design and analysis, image processing, signal processing and communication, financial modelling and analysis, etc. In addition, the Simulink is a supporting software package, which provides a visual development environment, it is often used in system simulation, dynamic/embedded system development and other aspects. Most of these toolboxes are written in open Matlab language. Users can not only view the source code but also modify and create the function according to their own needs. In addition, some users often publish their own programs or toolboxes in Matlab Central[1] for free download and use by other users.

2.1 Introduction of Matlab Interface

There are many versions of Matlab from early development to now. For my thesis, I used the version of 2014a.

When we double-click the icon of the Matlab, we will enter an interface like figure 2.1. The interface of Matlab can be divided into five parts: the menu, the command window, the current folder, the workspace and the command history. The positions and the sizes of these five parts can be changed according to the user’s own habits or needs. In addition to these main parts, other operation parts also can be moved to this interface.

[1] https://cn.mathworks.com/matlabcentral/

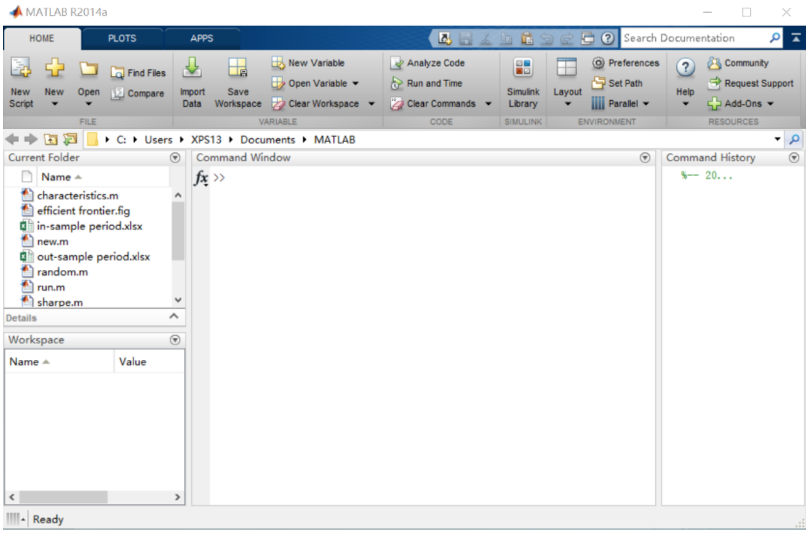


Figure 2.1 Interface of Matlab

2.1.1 Menu of Matlab

The menu is in the top of the Matlab interface, which is a basic operation guideline. From the figure 2.2, the menu consists of six parts, they are file, variable, code, Simulink, environment, and resources.

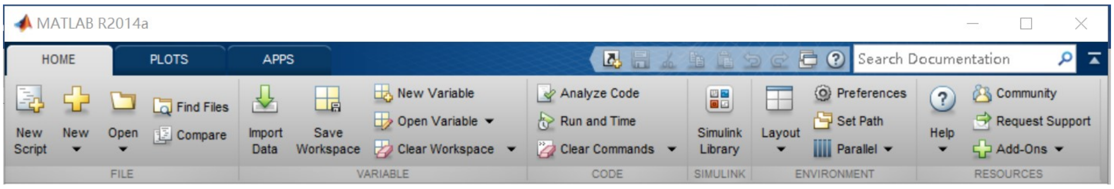


Figure 2.2 Menu of Matlab.

Under the file part, there are two common option: “New” and “Open”. The “New” option is mainly used for creating a new file, such as Script, Function, Figure and so on. Then, the “Open” option is mainly used for opening the existing files from the computer. In addition, there is a separate option, which is specially used for creating Script file. The reason is that the Script file is the most commonly used file type in the Matlab.

Under the variable part, there are two common option: “Import Data” and “Save Workspace”. The “Import Data” option is used for importing the data from the user’s computer, and we can choose the format of the data according to the user needs, such as table format, matrix format and so on. Then, the “Save Workspace” option is used for saving the history what we input.

And here are three other options, they are “New Variable”, “Open Variable”, “Clear Workspace”. Under the code part, the option of “Analyze Code” is used for displaying potential errors and problem. The option of “Run and Time” is used for helping you determine where you can modify your code to do a performance.

Then, about the Simulink Library, the details of this option are as the figure2.3 shows.

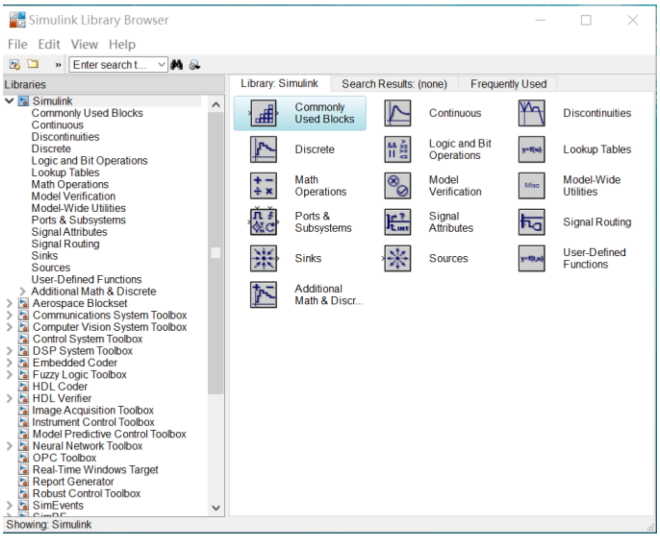


Figure 2.3 Simulink Library.

The Simulink was described earlier in this chapter, the Simulink is a visual simulation tool in Matlab. For the convenience of subsequent use, the user can create the new library to manage these models what they often use.

Under the environment and resources part, they are used for helping to manage the interface of Matlab and providing the help to user when they face the problem. For Example, the user can click the option of “help”, then they will enter a new interface, which is as figure2.4 shows. In this interface, we can see that there are many different toolboxes, and you can find any explanation of the function, model or theory what you want.

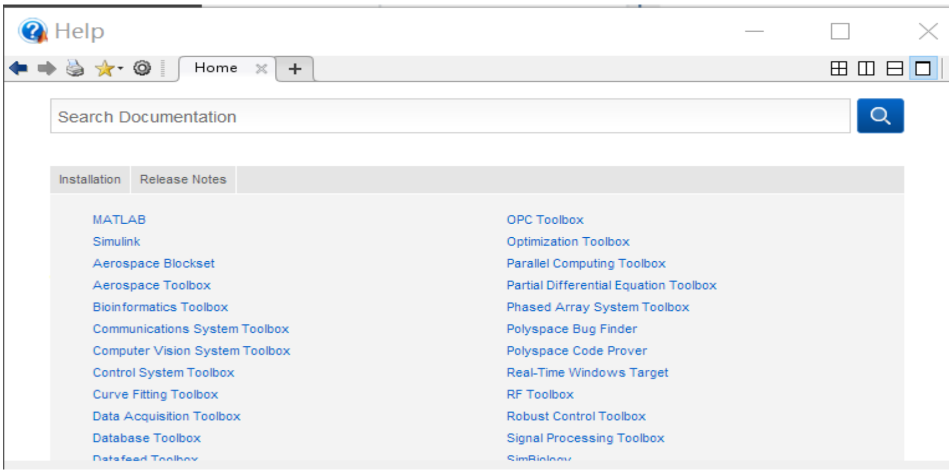


Figure 2.4 The interface of "help"

2.1.2 Command Window of Matlab

Command window is a window that is used for inputting the code and executing a command. It is in the center of Matlab interface. The command window also can save the result of calculation, it's good for users to check the records over and over again.

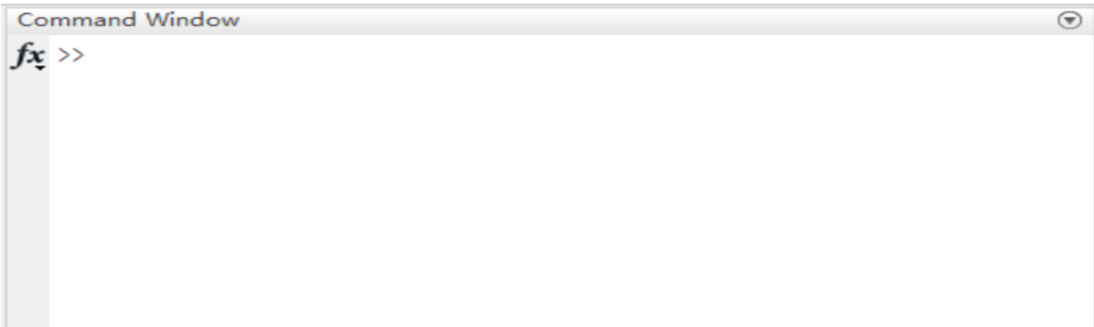


Figure 2.5 Command window.

As the figure2.5 shows, in the command window, ">>" is a command line prompt, which indicates the Matlab is in the ready status. You can input the code behind the command line prompt.

In the command window, the punctuation must be in English, and in most cases, space doesn't work. parentheses "()" represents the operation levels, square brakets "[]" is used to generate matrices, brace "{}" is used to compose the unit array. What's more, the function of semicolon ";" is that doesn't display the result of the operation, and the comma "," is used as the separator and used in district branches. In addition, the percent sign "%" is the symbol of note, the equality sign “=” is used for assignment and the double equality sign “==” represents the equivalent sign in mathematical.

Then, there are some common shortcut commands, which can be used in the command window.

Table 2.1 Common shortcut commands.

Clc	Clear the statement of command window
Clear	Clear the current workspace variables
Clear+name of variable	Clear the specified variable
Who	Display the list of current variables
Which+name of variable	Confirm whether the function in the current path
Save	Save the file in the computer
Help+name of function	Provide the definition for function
Ctrl+C	Stop the running program
↓ ↑	Switch to the command before or after

Source: own elaboration.

In the Matlab, these common shortcut commands will provide you with conveniently in your operation. For example, the shortcut key of “clc” will clear the all statements of command window when you don’t need these statements. At the same time, this shortcut key doesn’t clear the variable of current workspace, it also won’t eliminate your other records. In a word, if you can use the shortcut commands skillfully, it will save a lot of time for you.

2.1.3 Command History of Matlab

Command history is in the lower right of Matlab, which is used for showing the statement with currently and historically. The command history window can be undocked.

As the figure 2.6 shows, the time of historical operation can be displayed with green text in the command history, beyond that, if there is a mistake in your operation, it will be marked with a red line. The command history can be used not only to record historical operation information, but it can also be operated again. And the execution of the operation is implemented by the right mouse button menu.

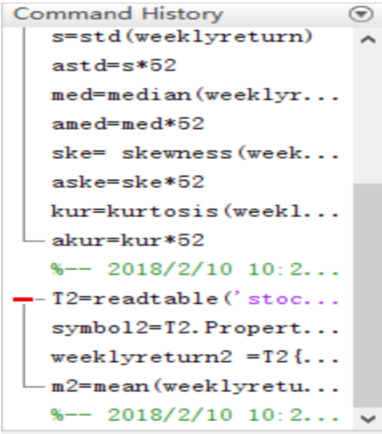


Figure 2.6 Command history of Matlab

2.1.4 Current Folder of Matlab

In the Matlab, the current folder is a browser, which can find some files in the current path.

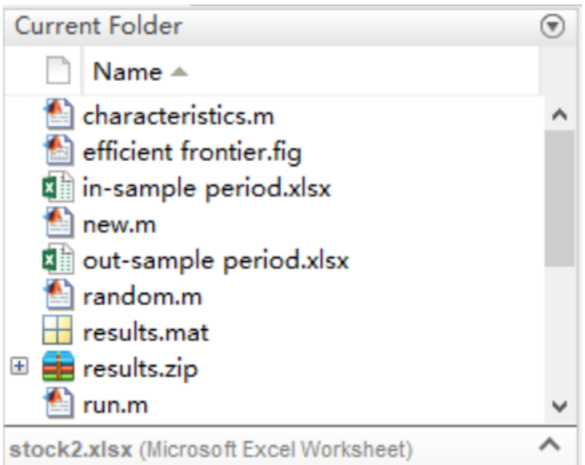
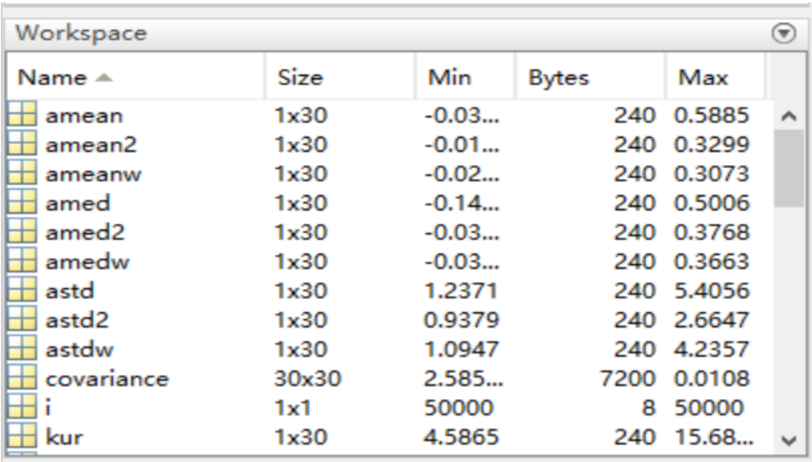


Figure 2.7 Current folder of Matlab.

From the figure 2.7, there are several types of documents that can be identified by Matlab: M file, excel file, mat file, zip file and figure file. These types of files are frequently used in Matlab, especially M file. With the help of current folder, the user can read the files what they load and save, and the user also can edit the file what they need. The current folder can help the user save the history of the file. When you open the Matlab next time, these files are still displayed in the current folder window. Of course, we also can set the current folder through the "preference menu", you can choose the number of the most recent folder to save in the current folder window, which the maximum number is 20. Besides, the user also can clear history.

2.1.5 Workspace of Matlab

The workspace is used for recording the variables what we calculate. According to figure 2.8, this window can display that the name, size, minimum value, maximum value and type of each variable. In this window, the variable can be deleted when the variable is not needed. What's more, the variable will be deleted after exit Matlab, but the user can save the variable in the Mat file.



Workspace

Name	Size	Min	Bytes	Max
amean	1x30	-0.03...	240	0.5885
amean2	1x30	-0.01...	240	0.3299
ameanw	1x30	-0.02...	240	0.3073
amed	1x30	-0.14...	240	0.5006
amed2	1x30	-0.03...	240	0.3768
amedw	1x30	-0.03...	240	0.3663
astd	1x30	1.2371	240	5.4056
astd2	1x30	0.9379	240	2.6647
astdw	1x30	1.0947	240	4.2357
covariance	30x30	2.585...	7200	0.0108
i	1x1	50000	8	50000
kur	1x30	4.5865	240	15.68...

Figure 2.8 Workspace of Matlab.

2.2 Matlab computing rules

As a mathematical software, Matlab is similar with Java and C/C++, meanwhile, matlab is strictly required for every symbol that is entered by the user, if there is a symbol that doesn't meet the requirement, Matlab won't be run.

There are three basic data types in Matlab: double precision array, cell array, and structure array. The data that the user input can be saved as these types according to the user's requirements. These rules[2] are as following:

a) The name of variable should be started with English character. The space can't be accepted, and number and other symbol should be used after the first English character.

2 http://www.doc88.com/p-4793075714930.html

b) User should distinguish the case of a letter, for example, *a* and *A* are different variables.

c) The length of variable name shouldn't longer than 31 characters.

2.2.1 M file

In addition to these basic rules of variables, there are some considerations of M file to take into account. Editor Debugger is programming window of Matlab, all extension of edit program are called M file[3], which can be divided into M function and M scripts, and the differences between them are as following:

(a) The M function file start with *function*.

(b) M function file generally has the variable of input or output, but the scripts don't show the variable of output.

(c) The variables of M function file are local variables, but variables in the M scripts are global variables, which are existing in command window.

(d) when we invoke the M function, we should execute the name of file in the command window, meanwhile, the variable of function should be assigned. However, when we invoke the M scripts, we can execute the name of file in the command window or select the option "Run" of the main menu to run.

Every time you modify the program, you have to save the M file again, the name of file should start with English letter, and can't be same with name of variable. The location of M file can be researched by code of "which" and inputting the command "type" will show the content of file.

2.2.2 The saving and invoking of data, commands and graphics

After we exit Matlab, the variable of command window will be lost. In order to save the value of variable, we can use the command of "save" to save the variable into the data file, its expansive name is MAT. MAT file can't be read, it needs to be invoked with "load" command. Besides, in order to exchange the data with other applications, "save" and "load" commands provide some different code to deliver the formatted data file, and the use of the format is

[3] https://wenku.baidu.com/view/43228b8dcc22bcd126ff0ccf.html

consistent with the C programming language. However, the need to pay attention to is that "save" command can only save variable and data and can't save the commands.

Then, in the window graphic, we can use the option of "save as" to save the graph as the fig file or M file, but it is important to note that the graph only can be opened in Matlab, we also can use the option of "File Export" to save the graph as the jpg file or bmp file. The most common way to save graphics is to use a graphical window menu to cut it as a picture into a Word file or other applications.

2.3 Program Structure of Matlab

In addition to executing numerical computation and symbolic operation by command, Matlab has a more important way of execution, called programming, which is similar to other high-level languages. The program structure of Matlab can be divided into three basic types: sequential structure, loop structure, and selective structure. Before we wrote the program, we should create a new M-file.

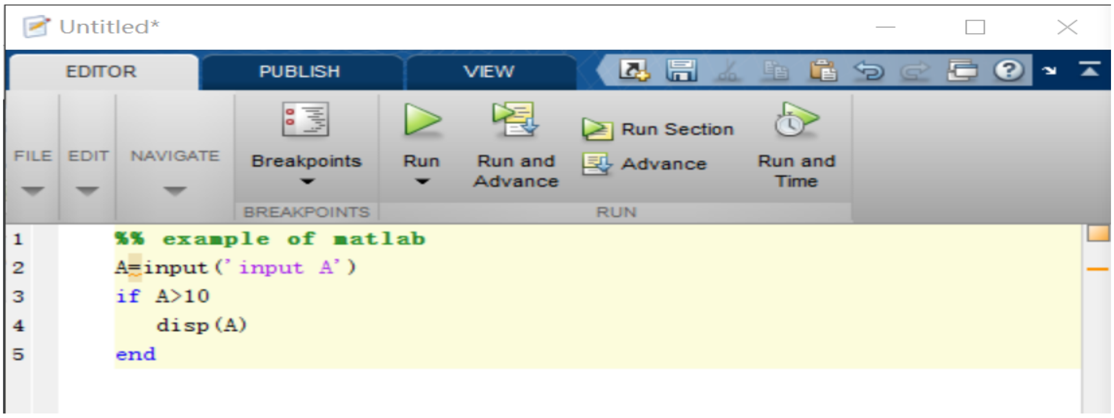


Figure 2.9 Example of Matlab.

As is shown in figure 2.9, it's an example of a kind of program. On the left of the program, the line number of the comments is displayed, and the text of program will be distinguished by different colors. The black color is used for the main part of a program, the green color is used for the note part of program, the purple is used for the attribute value of program, and the blue color is used for control statement, such as the statement of "if…end".

2.3.1 Sequential Structures

The sequential structure is arranging many single commands in sequence, and the Matlab will execute one by one from top to bottom according to the order of statements.

According to figure 2.10, this is a sample calculation, which is a kind of sequential structure. It means that the user input a command, the Matlab will execute according to the command.

```
>> A=3;
>> B=5;
>> C=A+B;
>> C

C =

     8
```

Figure 2.10 The example of sequential structure

Source: Own calculation.

2.3.2 Loop Structure

In the Matlab, the loop structure is used for executing the statements based on the conditions again and again. When user input a command, the Matlab will decide whether to run the loop according to the conditions.

There are two types of the loop structure, they are "for" loop and "while" loop. When the times of execution is determined, we will use the "for" loop structure. The figure 2.11 is shown that calculate the sum of 1 to 10 by using the "for" loop.

```
>> sumA=0;
for n=1:10
sumA=sumA+n;
end
>> sumA

sumA =

    55
```

Figure 2.11 The example of "for" loop structure.

Source: Own elaboration.

Then, “while” loop is used for the condition that the times of execution can’t be determined. And the sign that the program loop of executive termination is that the result of the logical expression is false or zero, otherwise the loop will continue to execute.

```
>> a=1
while a<3
    a=a+1
end

a =

     1


a =

     2


a =

     3
```

Figure 2.12 The example of “while” loop.

Source: Own elaboration.

This figure 2.12 is shown when the variable “a” is less than 3, we can get a result of “a+1” by using “while” loop.

2.3.3 Selective Structure

As the common structure of Matlab, the selective structure also can be called decision structure. It just like that the user asks a question, and the Matlab will take one or two actions according to the requirements.

There are two common selective structures, which are “if” structure and “switch” structure. The “if” structure can be divided into two types, which are “dual-alternative ifs” and “single-alternative ifs”.

From the figure 2.13, the condition of this program is that we should display “A” when the number of “A” is higher than 10. So, when we input the number 15, the number is displayed by Matlab.

```
>> A=input('input A');
if A>10
    disp(A)
end
input A15
    15
```

Figure 2.13 The example of single-alternative ifs.

Source: Own calculation.

```
>> x=input('x');
if x>10
    y=log(x);
else
    y=cos(x)
end
x3

y =

   -0.9900
```

Figure 2.14 the example of dual-alternative ifs.

Source: Own calculation.

According to the figure 2.14, the “dual-alternative ifs” is used for making a decision under the two options. If the number of “x” is higher than 10, we should execute the formula of “y=log(x)”, but if the number of “x” is not higher than 10, we should execute the formula of “y=cos(x)”. So, when we input the number 3, the Matlab executes the formula of “y=cos(x)”, the result is -0.9900.

```
>> month=input('please input month(1-12):');
switch month
    case{1,3,5,7,8,10,12}
        disp(31)
    case{4,6,9,11}
        disp(30)
    case 2
        disp(28)
end
please input month(1-12):2
    28
```

Figure 2.15 The example of switch structure.

Source: Own elaboration.

As can be shown in the figure 2.15, the switch structure is used for multi-branch selection of variables, each of these statements can also be added to another circular statement and branch statement. When the switch expression is matching the case, the Matlab will execute. If we input the statement is different from other cases, the Matlab will execute the statement of "otherwise".

2.4 Programs debugging

In the operating process, the Matlab will automatically recognize the mistake. The common mistakes show function and matrix.

The function mistakes are usually shown as a code that uses errors. The example is shown in the figure 2.16.

```
>> risk1=stdi(weeklyreturn)
Undefined function 'stdi' for input arguments of type 'double'.

Did you mean:
>> risk1=std(weeklyreturn)
```

Figure 2.16 The example of function mistake.

Source: own elaboration.

According to figure 2.16, we can see that Matlab will inform you with red words if you make mistake in function, of course, the wrong function can't be run. Meanwhile, the Matlab will provide the correct function for your choice below your function. Of course, if the function provided by Matlab is not meeting your condition, you can search function through the help center of Matlab.

The matrix mistakes are usually shown as the wrong computation and different dimensions. The example is shown in the figure 2.17. In this example, the matrix of "x" and "y" have different dimensions so that they can't be multiplied. In the circumstances we should check if there are same dimensions or check if there is a right process of calculation.

```
>> x=[2,3,4;6,7,8]
y=[2,6,7,8,9,0]

x =

     2     3     4
     6     7     8


y =

     2     6     7     8     9     0

>> x*y
Error using  *
Inner matrix dimensions must agree.
```

Figure 2.17 The example of matrix mistakes.

Source: own calculation.

2.5 Graphing in Matlab

Matlab has the ability with powerful numerical calculation, simultaneously has a convenient function of drawing, especially in visualizing all kinds of scientific computing structures, such as data and functions. In Matlab, the drawing of two-dimensional graphics is the basis of language processing graphics.

In Matlab, the most frequently used function in drawing graphs is the plot () function, which is generally called drawing function. In general, if we want to draw two more curves in the one figure, the curves should be distinguished by different colors and different types of line. Sometimes, in order to highlight some point of curve, we also need to mark the point. And in Matlab, we can use some commands to finish it. These commands are shown in table 2.2, 2.3 and 2.4.

Table 2.2 The commands for color.

Color	Commands	Color	Commands
red	r	yellow	y
green	g	black	k
blue	b	white	w

Source: https://cn.mathworks.com/help/matlab/ref/colorspec.html.

In addition to these fixed commands of color, the user also can adjust the color by themselves. For example, the color of grey doesn't have fixed commands, but we can change the proportion of "RGB" into "0.5 0.5 0.5".

Table 2.3 The commands for mark.

Command	Example	Command	Example
+	++++++	*	*******
.	……….	p	☆
o	ooooooo	d	◊
s	□	^	△

Source:https://cn.mathworks.com/help/matlab/matlab_prog/matlab-operators-and-special-characters.html.

Table 2.4 The commands for the type of curve.

Type	Command	Type	Command
Actual line	-	Colon line	;
Dotted line	--	Point line	.

Source:https://cn.mathworks.com/help/matlab/matlab_prog/matlab-operators-and-special-characters.html.

After these descriptions, there is an example for plotting in Matlab.

```
>> x=[0:0.1:2*pi];
y1=sin(x);
y2=sin(x*2);
y3=sin(x*3);
figure
plot(x,y1,'b:');
hold on
plot(x,y2,'r--');
hold on
plot(x,y3,'g-');
legend('sin(x)','sin(2x)','sin(3x)');
xlabel('x')
ylabel('y1,y2,y3')
```

Figure 2.16 The program for plotting in Matlab.

Source: own elaboration.

In the figure 2.16, the “pi” represents “π” . There need to mention that the function of “legend” is used for making the note for different lines so that the lines can be clearly identified. And we also can make the labels for the X-axis and Y-axis by the function of “xlabel” and “ylabel”. Then, when we finish these steps, the result as shown in figure 2.17.

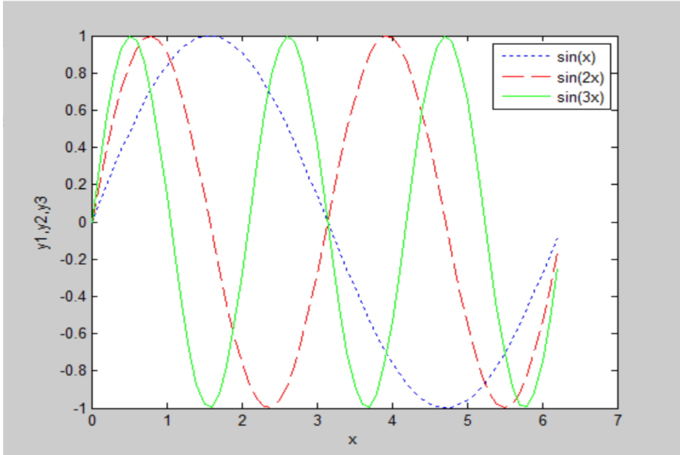


Figure 2.17 The example for plotting in Matlab

Source: own calculation.

3 Description of Portfolio Optimization Model

Portfolio optimization refers to restructure the investment to achieve the goal of spreading risk according to the specified target return and limited risk, which can apply probability theory, mathematical statistics, linear algebra and other relevant mathematical theories. It also can understand that the portfolio optimization reflects the specified risk level with maximization of return or a specified return with minimization of risk.

Some mathematical theories are mentioned in this chapter. In order to analyze data of stocks portfolio, we should know the basic information of stock portfolio by calculating in Matlab, such as the return of portfolio, mean of return, skewness of return, etc. These basic characteristics of stocks portfolio can help us analyze stock portfolios in more ways. Then, risk measures are introduced in next section, which is a way to analyze and predict risk to evaluate the possibility that the risk accident occurs and the loss that the accident may cause. Under risk measure, the standard deviation and Value at Risk(VaR) will be introduced. Subsequently, portfolio optimization under Markowitz mean-variable framework is applied in this thesis. The naive strategy also will be applied as a kind of method which calculates the asset's weight. After that, the performance measure will be used for analyzing the performance of stock portfolio.

Thus, this chapter can be divided into five parts, which are characters of stock portfolio, risk measure, naive strategy, Markowitz model, wealth path calculation and performance measure.

3.1 The basic characteristic of stock portfolio

In this part, we describe some basic characteristics of stock portfolio, such as return, mean of return, skewness of return, kurtosis of return, and standard deviation.

3.1.1 Return and mean of return of stock portfolio

Generally, the asset's return can be distinguished discrete and continuously compounded returns. In the thesis, the discrete return is used. In the discrete case, the return R_t is computed as a relative change of asset's price P_t, the formula is as follow:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}. \tag{3.1}$$

After calculating the return of assets, we should consider the return of stock portfolio that we choose. The formula is as follows:

$$R_{p,t} = \sum_{i=1}^{N} w_i . R_{i,t} \tag{3.2}$$

In this formula, the return of the portfolio $R_{p,t}$ is given as the sum of the products of specified return of stock $R_{i,t}$ and the weight of each stock w_i in the portfolio.

An investment portfolio will face the risk that can affect the actual return of investor. There is no way to accurately calculate actual return, but we can use the mean of return to replace it. Mean of return can be used by investors to calculate the rate of expected return of security. What's more, before a company manager decides if accept a certain investment, he/she can use it in capital budget. The formula is as follow:

$$\text{mean of return} = \frac{R_1 + R_2 + R_3 + R_4 + R_5 + \cdots R_n}{n}. \tag{3.3}$$

From the formula of mean of return, we can see that the mean of return is the simple mathematical average of a series of returns (R_n) generated over a period of time. The result can reflect the average return of each stock, but it's easy to find that the result of formula is easily affected by the extreme values. Generally, in the Matlab, the expected return can be computed through the function *mean(**R**),* the ***R*** is described as a matrix of the return.

3.1.2 Standard deviation and covariance of the return

In the financial market, the risk measurement is a primary focus, the traders and analysts try to use a lot of indexes to evaluate the volatility and risk of investment, but the most common index is standard deviation. Standard deviation is a statistical concept that represents the degree of dispersion, which has been widely used in the risk measurement of stocks and mutual funds. It is calculated based on the fluctuation of the net value over a period of time. Generally speaking, the higher the standard deviation, the greater the degree of risk. Mathematically, the standard deviation is the square root of the variance. So, the formula is as follow:

$$\mathrm{Var}(R_p) = \sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \, Cov(R_i, R_j)\,, \tag{3.4}$$

Where

$$Cov(R_i, R_j) = \rho_{ij}\sigma_i\sigma_j = \sigma_{ij}. \tag{3.5}$$

In the formula (3.5), ρ_{ij} means the correlation between the return i and return j, and the standard deviation of each asset (i and j) can be defined as σ_i and σ_j, what's more, the σ_{ij} and $Cov(R_i, R_j)$ represent the covariance between returns.

So, according to formula (3.4) and (3.5), we can get a new formula of variance,

$$\mathrm{Var}(R_p) = \sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \, \rho_{ij}\sigma_i\sigma_j \tag{3.6}$$

According to the previous formulas, we can get some further conclusions. We assume that the portfolio is composed of N assets, and the expected return of assets can be expressed as $\mathrm{E(R)} = \{\mathrm{E}(R_1), \dots, \mathrm{E}(R_N)\}^T$, and the covariance matrix of returns can be expressed as $\mathrm{Q} = \{\sigma_{ij}, i = 1, \dots, N, j = 1, \dots N\}$. In addition, the portfolio composition can be defined as $\mathrm{x} = \{x_1, \dots, x_N\}^T$.[4] After that, we can compute the expected return of portfolio $\mathrm{E}(R_p)$, and standard deviation σ_p, the formulas are as follow:

$$\mathrm{E}(R_p) = \textstyle\sum_{i=1}^{N} x_i \cdot E(R_i) = x^T \cdot E(R), \tag{3.7}$$

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i \cdot \sigma_{ij} \cdot w_j = x^T \cdot Q \cdot x\,, \tag{3.8}$$

$$\sigma_p = \sqrt{\sigma_p^2}. \tag{3.9}$$

In the Matlab, we can compute the variance of portfolio through function *var( )*, and function *std( )* is used for computing standard deviation.

In addition to these descriptions of standard deviation, there are some advantages of it. The

[4] Kresta (2015)

standard deviation is a best measure of variation, which is based on every item of the distribution. Then, it can be used for measuring the data distribution and less affected by some extreme value, moreover, it's possible to calculate the combined standard deviation of two or more group. But, there are disadvantage of standard deviation as well, it assumes that expected return is symmetric between the positive deviation and negative deviation, but in real life, the investor prefers to get the positive return and mainly focus on the analysis of loss.

3.1.3 Skewness and kurtosis of return

Skewness is a measure of the skew direction and degree of statistical data, which describes the characteristics of asymmetrical degree of statistical data distribution. So, if the return distribution of stock is right-skewed distribution, which usually appears as a left-leaning curve, it means that there is high return on this stock, by contrast, if the return distribution of stock is left-skewed distribution, which usually appears as a left-leaning curve, it means that there is low return on this stock. The formula is as follow:

$$S_k = \frac{E(X - EX)^3}{(VX)^{\frac{3}{2}}}. \tag{3.10}$$

In this formula (3.10), the S_k is skewness, the variance is used and is denoted by VX, and mean is denoted by EX. "A negative skewness $(S < 0)$ measure indicates that the distribution is skewed to the left; that is, compared to the right tail, the left tail is elongated. A positive skewness $(S > 0)$ measure indicates that the distribution is skewed to the right; that is, compared to the left tail, the right tail is elongated."[5]

In addition, we can obtain the value of skewness through the Matlab, which the function is *skewness( ).* The graph of skewness is shown in figure 3.1 and 3.2:

The kurtosis is similar to the skewness, which is a statistic that describes the degree of steep of the conceptual data distribution. And this statistic needs to be compared with the normal distribution. The formula is as follow:

$$\text{kurtosis} = \frac{E(X - EX)^4}{(VX)^2}, \tag{3.11}$$

[5] Rachev at al. (2005)

where the VX is variance, and EX represents mean. The kurtosis measure of normal distribution is 3. The value of kurtosis equals to 3, it indicates that there is no difference between the degree of steepness of overall data distribution and the normal distribution. If the value of kurtosis is higher than 3, then indicates that the distribution of overall data is steeper than normal distribution, “the tails of the symmetric nonmoral distribution are ‘thicker’ or ‘heavier’ than the normal distribution, which the probability distribution with this characteristic is said to be a ‘fat tailed’. When a distribution is less peaked than the normal distribution, it is said to be platykurtic. This distribution is characterized by less probability in the tails than the normal distribution. It will have a kurtosis that is less than 3 or, equivalently, an excess kurtosis that is negative.”[6] In the Matlab, the kurtosis can be calculated by function *kurtosis( )*.

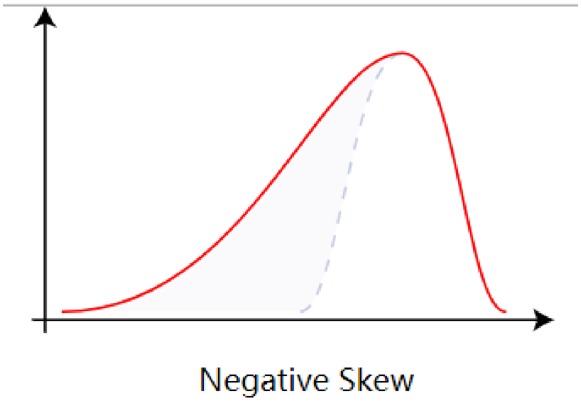


Figure 3.1 The example of negative skewness.
Source: Rachev at al. (2005)

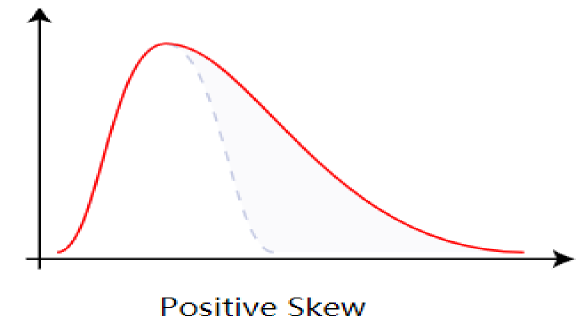


Figure 3.2 The example of positive skewness.
Source: Rachev at al. (2005)

[6] Rachev at al. (2005)

3.2 Risk measure

As a well-known word, risk has different explanations for different research aspects. In the financial sector, the financial risk refers to the uncertainty that the main participants of the financial market suffer in the market. So, ensuring to avoid and reduce the risk, risk measure can't be ignored. Risk measure is important for both individual investors and for financial companies, which is used for predicting and analyzing risk. We are able to calculate the probability of loss that can make the investors understand the consequences of the risk of the loss and focus on the consequences the risk brings.

The development of risk measure theory has experienced three stages: firstly, the variance and risk factor as the main measures are the traditional risk measure stage, the second is the modern risk measurement stage represented by the VaR, and the last one is the coherent risk measure that represented by Expected shortfall (ES for short). In this part, we will focus on the standard deviation and VaR, but now, the coherent risk measure will be briefly introduced. A risk measure $\rho(X)$ is defined as a mapping from a set of random variables to the real numbers, and the ρ must meet these requirements:

(a) Monotonicity: $X_1 < X_2,\ \rho(X_1) > \rho(X_2)$. The better investment portfolio, the lower risk.

(b) Positive homogeneity: $\forall \lambda > 0,\ \rho(\lambda X) = \lambda \rho(X)$. The number of assets increased by λ-times the risk of investment portfolio will increase by λ-times as well.

(c) Translation invariance: c is real number, $\rho(X + c) = \rho(X) - c$. If add the risk-free product or cash with c, the value of $\rho(X)$ will decrease by the same amount.

(d) Subadditivity: $\rho(X_1 + X_2) \le \rho(X_1) + \rho(X_2)$. The risk of two different investment portfolio can be combined to a new investment portfolio with a new risk, and the new risk is equal or lower than the sum of separate portfolio, which means that diversification of investment can spread risk.[7]

In this part, risk measure can be described in two aspects, which are standard deviation and Value at Risk. And the standard deviation was mentioned in previous chapter.

[7] Kresta (2015)

3.2.1 Value at Risk

Value at Risk has become a mainstream method to measure the market risk in financial field. We know that traditional Asset-Liability Management is too dependent on the report analysis to be short of timeliness, and it's too abstract to measure risk by variance, besides, the variance reflects only the fluctuation range. So, the value at risk model was proposed when these traditional methods are unable to accurately define and measure the risk.

Value at Risk (VaR for short) refers to the maximum possible loss of a financial asset or portfolio under a certain level of confidence in holding period. VaR is widely used risk measure, which is often applied in the bank and insurance companies (Basel and Solvency). It can be defined as follows,

$$VaR_{X,\alpha} = -\inf\{x \in R: F_X(X) \geq \alpha\}, \tag{3.12}$$

where F_X is cumulative distribution function, X refers to random-variable profit, it at time *t* can be computed as the wealth at time *t-1* times the random portfolio returns at time *t*. And α is the chosen probability level specifying the probability with which the observed loss can exceed estimated VaR. The value of α is 15%(Solvency II), 5% (original methodology proposed by JP Morgan), 1% (Basel II) and 0.5%(Solvency II), which can be chosen by investors. Generally, the choice of confidence intervals $(1-\alpha)$ reflects the different preferences of investor on risk.

$$\mathrm{P}_R\left(X \leq -VaR_{X,\alpha}\right) = \alpha \tag{3.13}$$

From the formula (3.13), if we want to determine the VaR of a portfolio, we must determine the following two coefficients:

(a) The value of α. Choosing different confidence intervals can reflect the different preferences of investor on risk.

(b) Holding period. We should calculate the maximum loss value of holding assets in any period of time.

(c) The observation periods. It's a time span of observation for the volatility and relevant return at a given period, we should balance between the possibility of historical data and the risk of structural changes in the market. In order to overcome the impact of business cycle, the longer period of historical data will be better. However, the longer the time is, the greater of the possibility of market structure change will be, the historical data will be difficult to reflect the condition of future.

3.2.2 Conditional Value at Risk

Conditional Value at Risk (CVaR for short) is also called expected shortfall, compared with VaR, this theory is a better kind of risk measurement, it can provide the answer to the question when the expected loss on portfolio over a given VaR. A lot of disadvantages exist in the VaR: firstly, VaR doesn't meet the rule of subadditivity, which means the portfolio risk may not lower or equal to the sum of risk on each asset, this phenomenon goes against the risk diversification phenomenon in the financial market; Secondly, VaR can't totally measure the expected loss on investment portfolio; So, under these conditions, the advantages of CVaR can't be ignored by investors. The CVaR can be defined as:

$$\mathrm{CVaR}_{X,\alpha} = -E[x | x < -VaR_{X,\alpha}], \tag{3.14}$$

where the X is a random-variable profit and x are the realizations of this random variable. Compared with VaR, CVaR considers the tail risk, it belongs to the measurement of risk subadditivity, what's more, the CVaR isn't easy to show the wrong information to mislead the investor, in addition to these advantages, based on the CVaR, the investment portfolio optimization is easier to implement.

If we assume that the profit X with equal probability, the CVaR will be defined as follows:

$$\mathrm{CVaR}_{X,\alpha} = -\frac{1}{\alpha}[\frac{1}{n}\sum_{\alpha=1}^{[\alpha n]-1} X_{\alpha} + [\alpha - \frac{[\alpha n]-1}{n}]X_{[\alpha n]}], \tag{3.15}$$

Where $[X]$ stands for the smallest integer larger than X and n is the quantity of data utilized for CVaR calculation[8].

[8] Kresta (2015)

3.3 Naive strategy

Naive strategy (Benartzai&Thaler, 2001) is a very common strategy for individual investor, this investment strategy is setting a fixed weight according to each type asset, and the weight doesn't change with time. Usually, the weight is determined by the investor preference. The 1/N weighted strategy is a kind of Naive strategy, which is an equal proportion of investment model, the formula is as follow:

$$w_i^{ew} = \frac{1}{N}, \tag{3.16}$$

Where the N is defined as number of assets. From this formula, we can see that this strategy doesn't consider other factors, such as return and risk.

3.4 Markowitz Mean-Variance model

Markowitz developed a theory of investment portfolio that can be operated under uncertain conditions, and that theory is mean-variance methodology. There are some assumptions[9] of that theory, which are as follow:

1. The analysis is based on single period model of investment. Thus, we don't allow the changes of portfolio structure during investment period.
2. An investor is risk averse and rational in nature.
3. The investor's utility function is concave and increasing due to his risk aversion and consumption preference.
4. Risk of the portfolio is based on the variability of returns from the portfolio.
5. We assume the efficient market without transaction costs and taxes.
6. Even infinitely small amount of money can be invested into the particular assets.

There exists the risk in the financial market, in order to reduce the risk, diversification of asset is very important for investor. In this model, investment's risk and return should be focused. As an investor, people are willing to get a maximum return at a given level risk, or they are willing to suffer the minimum risk at a given return. So, firstly, we should find the portfolio with maximal expected return, the objective function is as follow:

[9] Kresta (2015)

$$\mathrm{E}(R_p) \rightarrow max, \tag{3.17}$$

where

$$\mathrm{E}(R_p) = \sum_i x_i \cdot E(R_i) = \vec{x}^T \cdot E(\vec{R}). \tag{3.18}$$

In the formula (3.18), the weight of $i-th$ asset can be defined as x_i, and we can assume x = $[x_1, x_2, x_3 \dots \dots, x_n]^T$, the expected return on each asset is $E(R_i)$, the $E(\vec{R})$ can be defined as $E(\vec{R})$=$[E(R_1), E(R_2), E(R_3) \dots \dots, E(R_n)]^T$, and the expected return of portfolio is $E(R_p)$. After that, we constrain x_i equals or greater than 0, and the sum of x_i each portfolio equals to 1.

Secondly, the risk of portfolio should be considered, we should find the portfolio with minimal risk, the objective function is as follow:

$$\sigma_p \rightarrow min, \tag{3.19}$$

where

$$\sigma_p = \sqrt{\sum_i \sum_j x_i \cdot \sigma_{ij} \cdot x_j} = \sqrt{\vec{x}^T \cdot \boldsymbol{Q} \cdot \vec{x}}\,. \tag{3.20}$$

In this formula, $\boldsymbol{Q}$ is n × n covariance matrix, which is defined as $\boldsymbol{Q} = [\sigma_{ij}, i = 1,2, \dots$,n], σ_p is standard deviation of portfolio, also refers to risk, and the σ_{ij} is covariance between returns. Similarly, we constraint x_i equals or greater than 0, and the sum of x_i each portfolio equals to 1.

Then, to construct efficient frontier, we should find the efficient frontier points between maximum expected return and minimum standard deviation, the objective function is same as formula (3.19), which is $\sigma_p \rightarrow min.$ Under this condition, the constraints of efficient frontier are listed as shown in formula (3.21):

$$\begin{cases} \sigma_p \rightarrow min \\ x_i \geq 0, i = 1,2, \dots, n \\ \sum_{i=1}^{n} x_i = 1 \\ E(R_p) = R_{p-gen} \end{cases} \tag{3.21}$$

Where the R_{p-gen} is defined as specified initially for a given equidistant point, where

$$equidistant\ interval = \frac{E(R_{p-max}) - E(R_{p-min})}{n-1}. \quad (3.22)$$

In the formula (3.22), the $E(R_{p-max})$ is the expected return of maximum expected return portfolio, the $E(R_{p-min})$ is the expected return of minimum expected return portfolio, and the n means the number of the portfolio.

In addition to these operations of computation, we can do this model through Matlab. Firstly, we can use the function $Portfolio(\,)$ to set up a portfolio according to the stock data what we invest, then, we can input the code based on the formula (3.1) to calculate the return of each asset, for example, we can input the code like: *weeklyreturn=T{2:end,2 : end}./T{1:end-1,2 : end}-1,* where the T is a symbol of creating table from file. After that, we should calculate the mean, covariance and variance for the return, the function of Matlab is as shown in formula (3.23):

$$\begin{cases} me = mean(\,) \\ covariance = cov\,(\,). \\ variance = var\,(\,) \end{cases} \quad (3.23)$$

Subsequently, these constraints of this model can be set in Matlab, which the code is as shown in formula (3.24):

$$\begin{cases} p = estimateAssetMoment(\,) \\ p = setDefaultContraints(\,), \\ p = setAssetMoment(\,) \end{cases} \quad (3.24)$$

Where the function of $estimateAssetMoment$ is used for estimating the mean and covariance of asset return from data, and the function of $setDefaultConstraints$ is used for setting up the portfolio constraints with non-negative weight that sum up to 1. Then, the function of $setAssetMoment$ is defined as setting moments of assets return.

Moreover, if we want to determine the weight, these functions need to use in the Matlab, they are listed as follow:

$$\begin{cases} ws = estimateMaxSharpeRatio(p) \\ we = estimateFrontier(p) \end{cases}, \quad (3.25)$$

Where the function of $estimateMaxSharpeRatio$ is used for estimate efficient portfolio that maximizes the Sharpe ratio[10], and the function of $EstimateFrontier$ is used for estimating the specified number of optimal portfolios over entire efficient frontier, which is transfer from formula (3.21). After all the calculations, we can get the graph of efficient frontier through function $plotFrontier$ in the Matlab, the example is as shown in figure 3,1.

Besides these two kinds of weight, the random-weight portfolio is created by the function $rand(\,)$, after that, the “for” loop can be applied to get the weights in percentages and the sum equal to 100%, the program is shown in grogram 3.1:

Program 3.1 The process of generating random weights in Matlab.

```
%% Genarate random weights
wr=rand(30,50000);
for i=1:size(wr,2)
    wr(:,i)=wr(:,i)./sum(wr(:,i));
end
```

Sources: own elaboration.

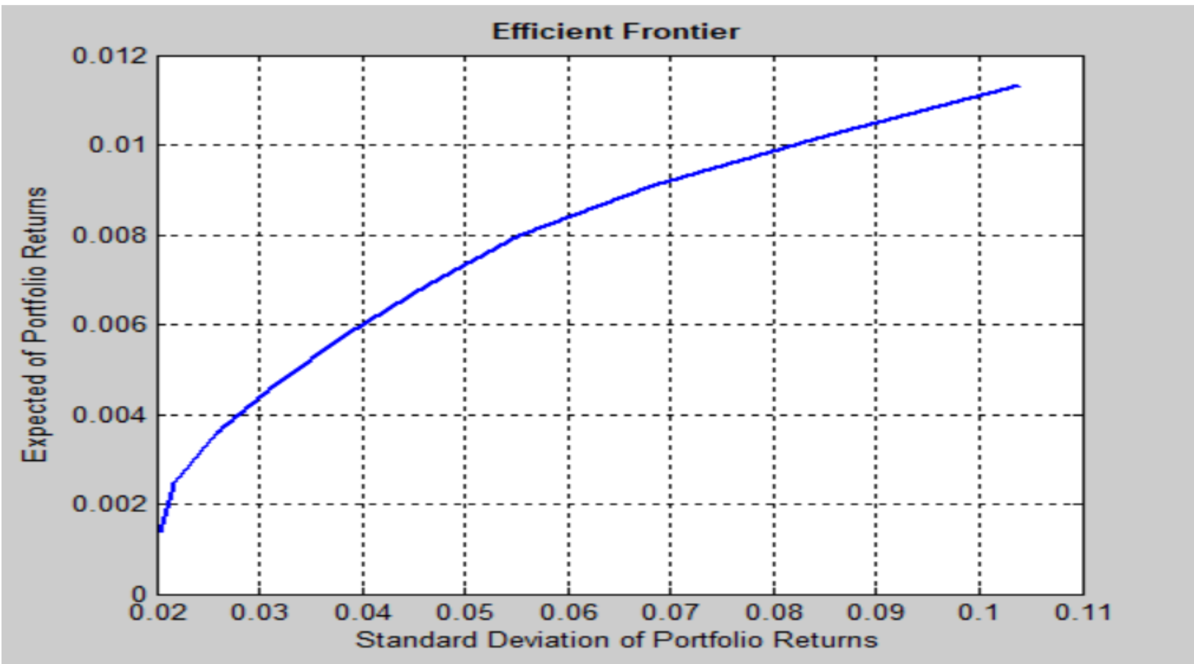


Figure 3.3 The example of the efficient frontier.

Source: own calculation.

[10] Sharpe ratio is described in 3.5.

3.4 Wealth path calculation

After previously descriptions of formula, we can compute the ex-post portfolio return, which is as follow:

$$\mathrm{R}_{p,t} = R_t \cdot w_t, \tag{3.26}$$

Where the R_t is observed returns, it can be defined as $R_t = \{R_{1,t}; R_{2,t}; \dots\dots; R_{n,t}\}^T$ and w_t is weights of assets in portfolio, which were obtained by portfolio optimization based on the return/prices of the assets[11], it can be defined as $w_t = \{w_{1,t}; w_{2,t}; \dots\dots; w_{n,t}\}^T$. Then, we can compute ex-post wealth evolution,

$$W_{t+1} = W_t \cdot \left(1 + R_{p,t}\right). \tag{3.27}$$

The variable of W_t is the wealth of initial investment, and the W_{t+1} is the wealth that after the investor get the return from the initial investment. It also can compute the wealth in Matlab, we can input these functions, which are as shown in formula (3.28):

$$wealth = cumprod(1 + return\ ), \tag{3.28}$$

3.5 Performance measures

Performance measurement is applied to measure the performance of the investment portfolio from the aspect of risk and expected return. In my thesis, the sample period of stock portfolio will be divided into in-of-sample period and out-of-sample period, we will calculate the same indexes of out-of-sample period based on the results of in-sample period, then, according to the results of out-of-sample period, we can know the performance of stock portfolios. In this part, we will introduce the performance measures what will be used, which are out sample return, out sample standard deviation, out sample variance, Sharpe ratio and maximum drawdown.

[11] Kresta (2015)

3.5.1 Out-of-sample return of stock portfolio

Out-sample return is an important measure to evaluate the performance of stock portfolio. We can know the how the return is during the out-of-sample period if we continue to stick to the decision of in-sample period.

The out-sample return can be calculated according to the equation (3.2). In the previous chapters, we know that can obtain the weight of in-sample period of stock portfolio from mean-variance model, then, in order to evaluate the return of out-sample period, we will calculate the return with the weight of in-sample period, the formula will be defined as follow:

$$R_{out-p} = \sum_{t=1}^{N} w_{in} \cdot R_{out,i} \tag{3.29}$$

The weight of in-sample period is defined as w_{in} in equation (3.29), and the $R_{out,i}$ represents the return of each asset in out-sample period.

After we calculate the out-sample return of stock portfolio, we can compare with in-sample return of stock portfolio. Of course, the higher return, the better for investor.

3.5.2 Out-of-sample standard deviation and out-sample variance

In fact, evaluation of stock portfolio can't use only return as an indicator, risk is an important indicator for a stock portfolio as well. In financial market, standard deviation is the most common index for risk, meanwhile, the standard deviation is the square root of the variance.

So, in order to evaluate the stock portfolio in out-sample period, we should compute the variance and standard deviation of out-sample period, which the formulas are same with equation (3.8) and equation (3.9). Compared with standard deviation of in-sample period, calculating the standard deviation of out-sample period should use the data of out-sample period. After that, we can compare the risk of each stock with different period.

Then, we need to consider both risk (standard deviation and variance) and return for a stock portfolio. All investors want to have a stock portfolio, which has the higher return at a given risk or has a lower risk at a given return. At this time, we can evaluate the stock portfolio according to the result of these two indicators of out-of-sample period.

3.5.3 Sharpe Ratio

Sharpe ratio is method to measure the performance of an investment by adjusting for its risk. In the investment activities, the investor prefers to endure higher risk of fluctuating when there is higher expected return, so, the main idea of the Sharpe ration is: for the rational investor, the main gold of the chosen investment portfolio is that seek maximum expected return at a fixed risk or seek the lowest risk under fixed expected return. The formula of Sharpe ratio is can be defined:

$$\text{Sharpe ratio} = \frac{\left[E\left(R_p\right) - R_f\right]}{\sigma_p}, \tag{3.30}$$

where the $E\left(R_p\right)$ is expected return of portfolio, R_f is risk-free rate and σ_p is standard deviation of portfolio. From the formula (3.30), the formula also can be understood as the percentage between the investment return and how much risk investor bear. Thus, the higher ratio, the better for investment portfolio.

3.5.4 Maximum Drawdown

Maximum drawdown, the maximum loss from a peak to a thorough of a portfolio, is an indicator of downside risk over specified time period. "if we assume wealth path $W_{(t)}$, we can measure the decline from the past maximal peak at time t."[12] This measure is called drawdown and can be expressed in formula (3.31):

$$DD_t = 1 - \frac{W_{(t)}}{\max\limits_{t \in (0,t)} W_{(t)}}. \tag{3.31}$$

Then, we can extend the ratio so that we measure the maximum drawdown over the period $(0, T)$,

$$MDD_{0,T} = \max_{t \in (0,t)} \left(1 - \frac{W_{(t)}}{\max\limits_{t \in (0,t)} W_{(t)}} \right). \tag{3.32}$$

The maximum drawdown is the worst decline in the wealth over analyzed period.

[12] Kresta (2015)

4 Calculation of portfolio optimization in Matlab

The basic knowledge of Matlab and several theories of portfolio optimization are introduced in previous two chapters. In this chapter, we apply these theories to solve the actual problem with actual stock data by applying the Matlab. We choose 30 stocks from—components of Dow Jones Industrial Average from the year 2008 to the year 2018, and there are four different models we apply to solve the problem on the portfolio optimization, which are naive strategy, Markowitz mean-variance model, random model and the max Sharpe ratio strategy. Beyond that, the sample period of this stock portfolio is divided into in-sample period (17/3/2008-31/12/2012) and the out-of-sample period (7/1/2013-15/1/2018). We use the 1 dollar as an initial wealth, and calculate the wealth path under each model, after that, we compare the performances in in-sample period and out-of-sample period.

This chapter can be divided into four parts, the first part is used for introducing the characteristics of sample data, the second part is used for making an analysis of the performance in the in-sample period under four different models that we have mentioned before, and the third part is used for analyzing the performance in the out-of-sample period under four models. The last part is making comparison between two periods under four models.

4.1 Data description

In this thesis, the source of portfolio of stock data is from Yahoo Finance[13]. We choose 30 stocks from the Dow Jones Industrial Average index during past ten years, these 30 stocks are usually keeping the position of top 30 with their performance. These data are presented in the form of weekly data of stock adjusted close prices, so, there are approximately 514 week's stock price data. The stock price is shown in dollar. The list of the stock names we choose is shown in table 4.1.

[13] https://finance.yahoo.com/

Table 4.1 List of company's name and abbreviations.

Name	Abbreviations	Name	Abbreviations
Apple Inc.	AAPL	McDonald's Corporation	MCD
American Express Company	AXP	3M Company	MMM
The Boeing Company	BA	Merck&Co, Inc	MRK
Caterpillar inc.	CAT	Microsoft Corporation	MSFT
Cisco Systems	CSCO	NIKE,Inc.	NKE
Chevron Corporation	CVX	Pfizer Inc.	PFE
Dillard's, Ins.	DDS	The Procter&Gamble Company	PG
The Walt Disney Company	DIS	The Travelers Companies,Inc.	TRV
General Electric Company	GE	UnitedHealth Group Incorporated	UNH
The Goldman Sachs Group,inc.	GS	United Technologies Corporation	UTX
The Home Depot, inc.	HD	Visa Inc.	V
International Business Machines Corporation	IBM	Verizon Communications Inc.	VZ
Intel Corporation	INTC	Walmart Inc.	WMT
Johnson&Johnson	JNJ	Exxon Mobil Corporation	XOM
JPMorgan Chase&Co.	JPM	The Coca-Cola Company	KO

Source: https://finance.yahoo.com/

Dow Jones Industrial Average (DJIA for short) is one of several stock market indices, created by Wall Street Journal editor and Dow Jones& Company co-founder Charles Dow, which is one of oldest and most credible American market indices. At present, The DJIA is consisting of 30 large companies, which are the representative of American blue chip. More and more people think DJIA is not an ordinary financial indicator, but a symbol of the world's financial culture, the first reason is that the stocks selected by the DJIA are all representative, and these companies who issue these stocks have significant influence in the industry, and their performances are attracting the attention of the world stock market. The second reason is that the average stock price index has never stopped to develop so that can be used for comparing the condition of stock market in different periods as well as sensitively reflecting the change of American stock market.

Table 4.2 The mean of return and standard deviation of chosen stock (Annualized)

	whole period		in-sample period		out-of-sample period	
	mean of return	standard deviation	mean of return	standard deviation	mean of return	standard deviation
APPL	27.5556%	30.2715%	34.8971%	35.1479%	20.5504%	24.7709%
AXP	13.9189%	35.3230%	16.1955%	46.6081%	11.7465%	19.2409%
BA	20.5871%	29.9990%	7.5878%	36.7892%	32.9910%	21.5559%
CAT	13.9424%	33.4705%	13.9914%	41.9527%	13.8957%	22.6765%
CSCO	9.0085%	27.5578%	1.5567%	32.9522%	16.1190%	21.1881%
CVX	7.7369%	24.7479%	10.3630%	29.4072%	5.2311%	19.3348%
DDS	30.7320%	58.7380%	58.8453%	74.9617%	3.9063%	36.9532%
DIS	16.3440%	25.4682%	15.1691%	31.2636%	17.4650%	18.3685%
GE	-2.7644%	31.9550%	-3.6594%	40.9485%	-1.9104%	19.9873%
GS	10.7728%	39.2905%	6.7847%	51.6604%	14.5781%	21.8119%
HD	23.3443%	26.4906%	22.5061%	33.8077%	24.1441%	16.8372%
IBM	5.9826%	21.9267%	13.3292%	24.6547%	-1.0275%	18.9591%
INTC	10.2661%	26.2811%	4.0673%	30.7855%	16.1809%	21.1360%
JNJ	9.2800%	15.1810%	3.3623%	17.1554%	14.9267%	13.0068%
JPM	16.8264%	40.1123%	13.8030%	53.8466%	19.7113%	19.5886%
KO	6.0232%	16.9512%	6.4173%	19.9001%	5.6472%	13.5928%
MCD	12.9978%	16.3477%	12.1997%	18.8194%	13.7593%	13.6133%
MMM	13.8341%	20.7833%	7.4496%	25.5129%	19.9261%	14.9411%
MRK	6.1632%	23.7558%	3.5501%	29.0681%	8.6566%	17.2783%
MSFT	14.6775%	25.4144%	2.3204%	28.7100%	26.4686%	21.7415%
NKE	17.0744%	26.6939%	14.4613%	31.7432%	19.5678%	20.8219%
PFE	8.1522%	21.6075%	8.2066%	25.7831%	8.1004%	16.7298%
PG	4.0941%	16.2622%	1.7389%	19.0065%	6.3415%	13.1506%
TRV	13.2578%	23.1429%	13.5405%	29.8222%	12.9881%	14.1469%
UNH	25.2884%	34.8610%	18.5876%	46.2165%	31.6822%	18.4385%
UTX	9.3360%	22.7179%	8.0346%	27.2094%	10.5779%	17.4379%
V	23.7670%	26.2881%	24.0262%	33.1890%	23.5197%	17.3821%
VZ	6.5457%	19.9544%	8.2509%	22.8555%	4.9187%	16.7664%
WMT	8.4201%	18.2367%	7.3608%	19.5901%	9.4308%	16.8810%
XOM	2.3005%	19.9673%	3.8145%	23.7435%	0.8558%	15.5776%

Source: own calculation.

After calculation, the results of mean of return and standard deviation are shown in table 4.2. To distinguish different character for each value, we use different color to represent them: the greener color represents the better data. For example, under the mean of return, the higher value with greener color represents the higher return we get from this stock, but under the standard deviation, the lower value with greener color represents the lower level of risk we suffer from this stock. According to table 4.1, under the whole period, DDS has the highest mean of return as well as the highest standard deviation (risk), and the same performance also shows in in-sample period, however, in the out-of-sample period, DDS has the highest standard deviation but relatively lower mean of return. In the out-of-sample period, BA has the highest mean of return, and MCD has the lowest standard deviation.

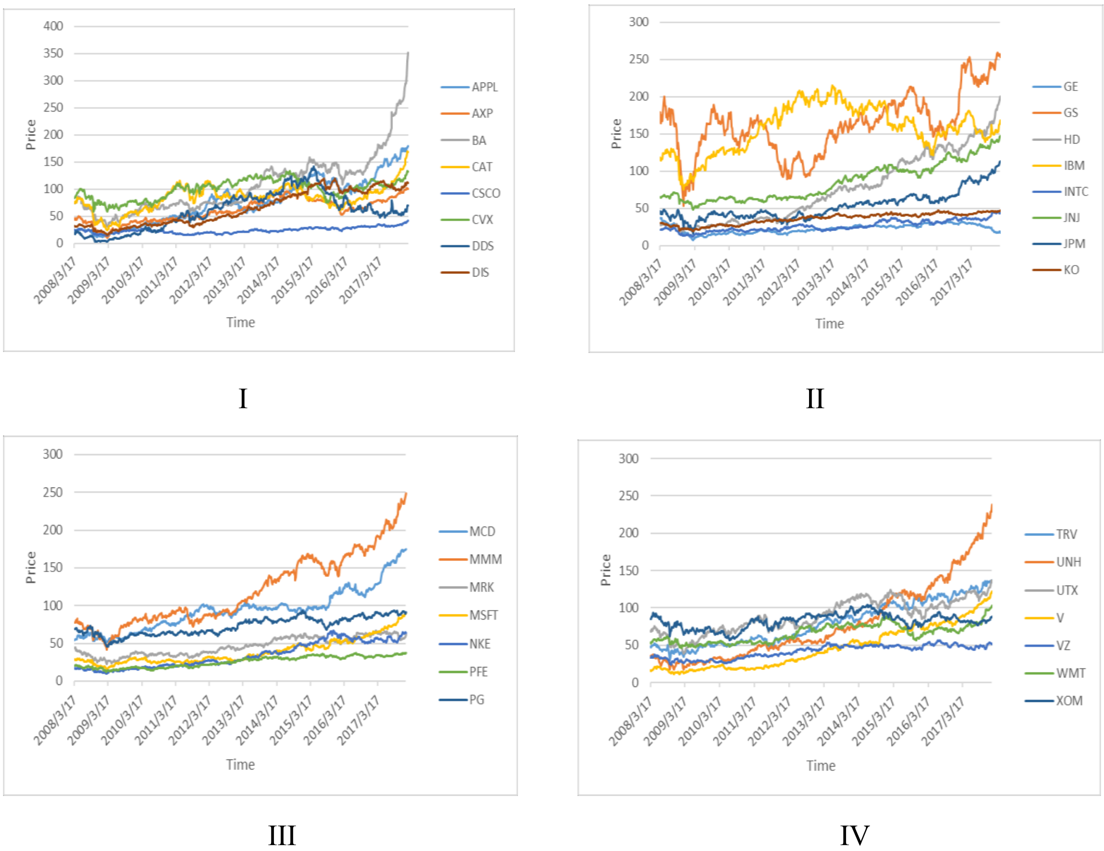


Figure 4.1 Evolution of chosen stocks.

Source: own calculation.

From the figure 4,1, there are four graphs, and each graph has ten curves that represent different stocks with different colors. These four graphs represent the change of stock price of 30 companies from 2008 to 2018. It's easy to find that there is a common point for these 30 curves, their stocks price all had a decreasing during the year of 2008, especially GS (the Goldman Sachs Group), which the stock price fell 50 dollars from 200 dollars during the year of 2008 to 2009. In 2008, "global financial markets have been buffeted by a series of extraordinary and tumultuous events. Most acutely that past fall, a global contagion of fear and panic choked off the arteries of finance, compounding a broader deterioration of the global economy"[14]. The stock price of GS had a huge decreasing because of American financial crisis in 2008. The business of Goldman Sachs Group covers investment bank, securities trading and wealth management. From the annual report of GS, we can know that its net revenues fell 52% to $22.2 billion and net earnings decreased 80% to $2.3 billion in 2008. Just because of its high coverage

[14]http://www.goldmansachs.com/investor-relations/financials/archived/annual-reports/2008-entire-annual-report.pdf

financial business in the financial market that led to the heavier damage on the financial condition of company in financial crisis.

In the graph I, the BA all kept the trend increasing in recent ten years, especially in recent three years, which the stock price had a huge increasing, it was from 100 dollars rising up to near 350 dollars. BA (the Boeing company) is the leader of the world's aerospace industry as well as one of largest manufacturers of civil and military aircraft in the world. As the main service provider of NASA, the BA operates both international space station and space shuttle. Moreover, it also provides many kinds of support service for military and civil airline. As far as sales are concerned, the BA is one of largest exporters in the America. In recent years, the BA has been obtained a sound development, which benefit from the long-term increasing trend of global air transport and the increasing demand of emerging nations. And from the BA's annual report of 2017, we know the company has "achieved $93.4 billion in revenue from record commercial airplane deliveries, strong services growth and solid performance in defense business"[15]in that year. Thus, such a good financial condition is enough to make its stock price have a huge increasing.

In graph II, IBM's stock price has been fluctuating when other stocks are basically rising. It's easy to find that IBM's stock price kept increasing from 2009 to 2013, but the stock price began to decline since 2013. "IBM manufactures and markets computer hardware, middleware and software, and provides hosting and consulting services in areas ranging from mainframe computers to nanotechnology"[16]. In 2009, the financial crisis was not completely over, it still has influence on the financial market, but the IBM began to recover the financial condition of company, and in 2010, IBM is well positioned to grow as the global economy recovers, there are many opportunities to develop the business, at that time, the demand of hardware market had not yet reached saturation so that the company has a lot of space for development. But with the development of IT industry, the demand of hardware market has reached saturation, and many other IT companies have gradually developed, which led to a decline in corporate revenue. When the revenue falls, the stock price fall with it.

In graph III and graph IV, we can see that the chosen 14 stocks are all with an increase tendency.

[15] http://s2.q4cdn.com/661678649/files/doc_financials/annual/2017/2017-Annual-Report.pdf
[16] https://en.wikipedia.org/wiki/IBM

The highest growth among these stocks is from the UNH (UnitedHealth Group Incorporated) and the MMM (3M company). UNH is the largest healthcare company in the world, and also the largest provider of corporate health insurance in the United States. The business of UNH can be divided into health insurance and health service, which are independence from each other but close conjunction with each other. Health insurance provide the service to employee of company, old people and the people who has low income. The health service focus on the responsibilities of health, medical care, nursing, and so on. It can not only meet the needs of service of insurance business, but also be able to get the individual customers. With the development of technology, UNH tried to perfect its service and help people get more benefit, of course, the company also obtain the profit from these services, from the annual report of UNH, in 2016, "Consolidated revenues increased by 18%, UnitedHealthcare revenues increased 13% and Optum revenues grew 24%", and in 2017, "Consolidated revenues increased by 9%, UnitedHealthcare revenues increased 10% and Optum revenues grew 9%"[17].

We have mentioned the characters of skewness in previous chapter, so we know that if the value of skewness is negative, the distribution is skewed to left, however, if the value of skewness is positive, the distribution is skewed to right. And if there exists the large absolute value of skewness, which indicates there are extreme values in this sample data.

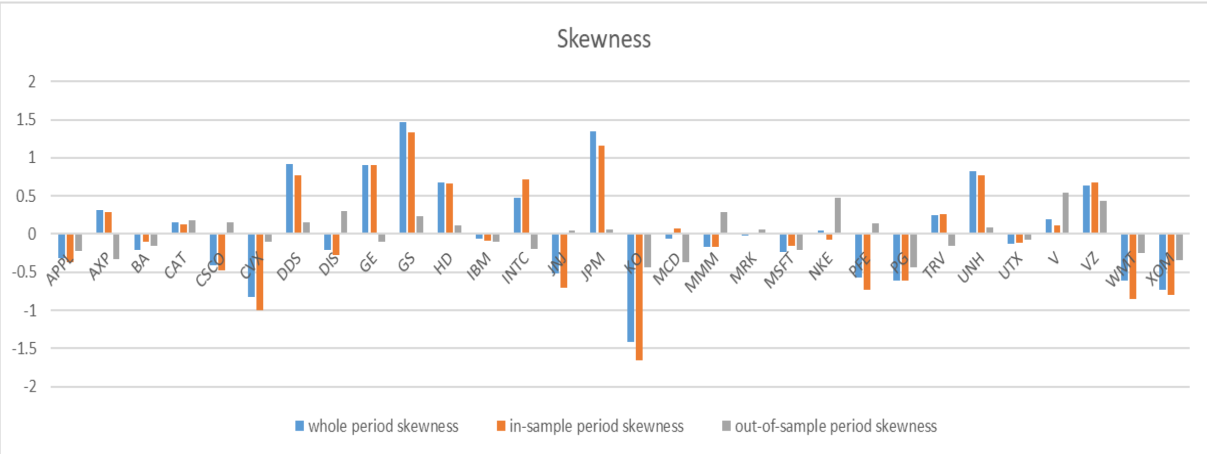


Figure 4.2 Skewness of chosen stocks.

Source: own calculation.

[17] http://www.unitedhealthgroup.com/~/media/D0B33E36E5C248ACB8B0F2BF9658B2FB.ashx

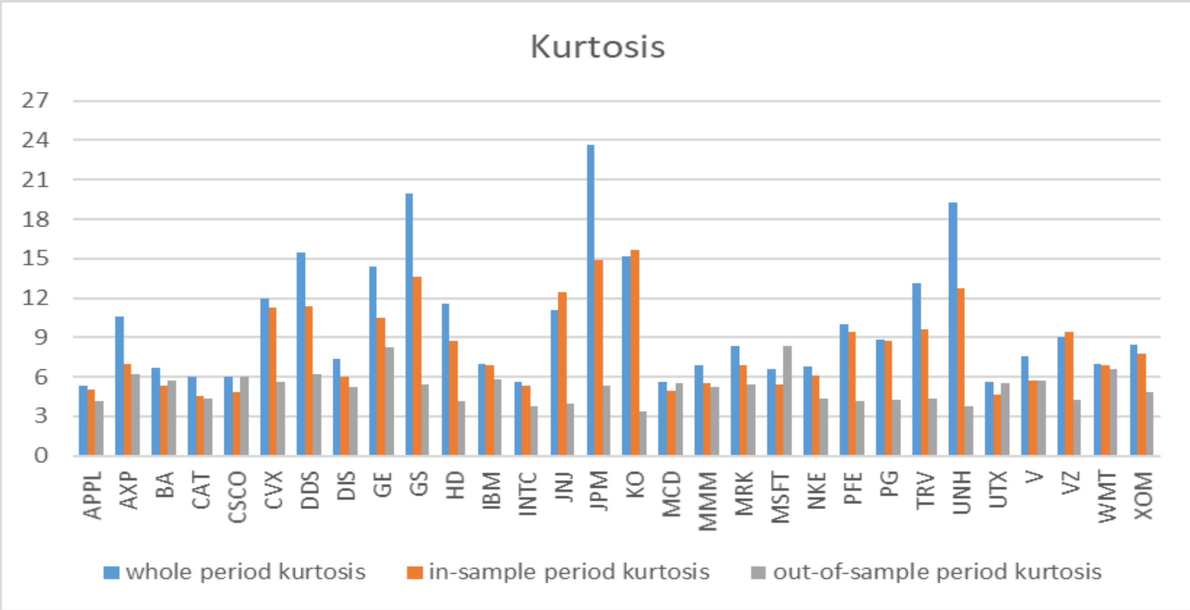


Figure 4.3 Kurtosis of chosen stocks.

Source: own calculation.

According to figure 4.2, we can see that there exist some stocks with large absolute value of skewness in whole-sample period and in-sample period, such as GS, JPM and KO. For the GS and JPM, their value of skewness is positive in whole-sample period and in-sample period, so the distributions are skewed to right, it indicates that there are some minimal values to affect the sample data, but in the out-of-sample period of them, the values of skewness are near zero, it means that the distributions are close to the normal distribution. For the KO, its value of skewness is negative in these three samples period, so the distributions are skewed to left, which indicates that there are some maximal values to affect the sample data.

According to the figure 4.3, It's easy to find that the values of kurtosis are all higher than 3, it means that the distributions of chosen stocks are all steeper than normal distribution, it also refers to that there exists the heavy tails, what's more, there are some extreme value in these kinds of sample periods, such as GS, JPM and KO. Then, from figure 4.2 and 4.3, we can find that the skewness and kurtosis are closely interconnected. For example, we have mentioned that the GS, JPM and KO have some extreme values, they are also with large absolute value of skewness, so, in the figure 4.3, the GS, JPM and KO are also with large value of kurtosis as well as with steeper distribution.

Before other calculations, we should determine the weekly risk-free risk rate. We can get the 10-year government bond rate of America that equal to 2.813%[18], so, the annual risk-free rate is 2.813%.

4.2 The application of models in the in-sample period

In this part, we will calculate all the models in the in-sample period what we describe in the previous chapter. They are naive strategy, Markowitz model, random model and max Sharpe ratio model.

4.2.1 Naive strategy

Under the naive strategy, no matter how the changes in stock price, the weight of each stock will not change, furthermore, the weight of each stock in investment is the same. It also means the investor will invest same stock at same relative weight in a portfolio.

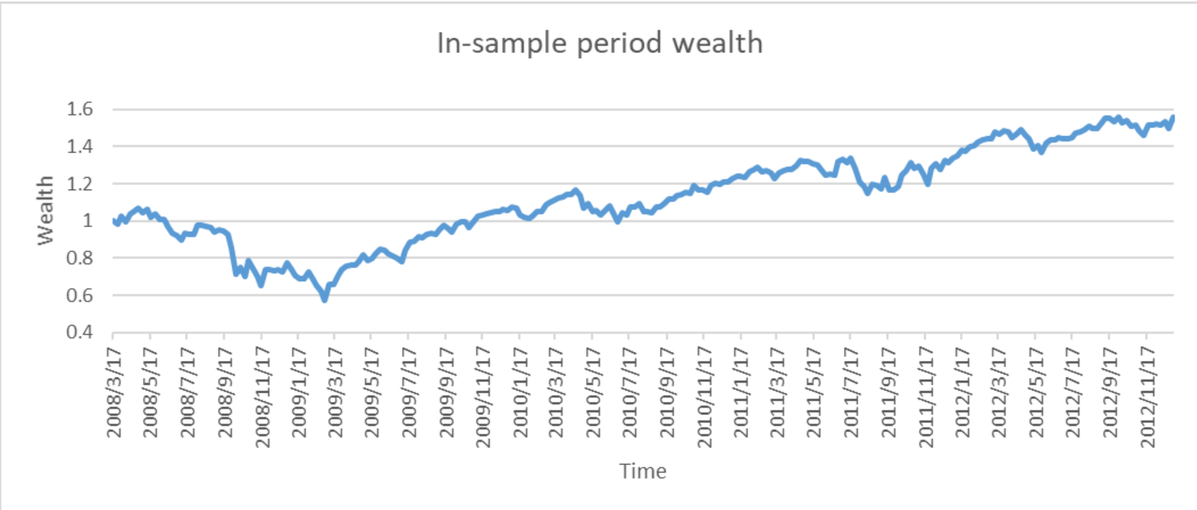


Figure 4.4 The trend of in-sample period wealth under naive strategy.
Source: own calculation.

In this thesis, the investment portfolio contains 30 stocks from 30 different company, so the weight of each stock is $\frac{1}{30}$. We can calculate the return of portfolio and the wealth in each week according to the formula (3.1) and (3.28), after that, we can obtain the in-sample period

[18] https://cn.investing.com/rates-bonds/u.s.-10-year-bond-yield

portfolio return and wealth under naive strategy, the result is shown in table 4.2 and figure 4.4.

Table 4.2 In-sample period portfolio return and wealth under naive strategy (in dollar).

Date	Portfolio Return	Wealth	Date	Portfolio Return	Wealth
2008/3/17		1	2010/8/16	-0.00372	1.04851
2008/3/24	-0.01678	0.98322	2010/8/23	-0.00712	1.04104
2008/3/31	0.04097	1.02350	2010/8/30	0.03303	1.07542
2008/4/7	-0.02511	0.99780	2010/9/6	0.00164	1.07719
2008/4/14	0.04161	1.03932	2010/9/13	0.01672	1.09519
2008/4/21	0.01136	1.05112	……	……	……
……	……	……	2011/5/2	-0.00725	1.31660
2008/12/29	0.06786	0.77613	2011/5/9	0.00147	1.31853
2009/1/5	-0.04507	0.74115	2011/5/16	-0.00852	1.30730
2009/1/12	-0.04595	0.70710	2011/5/23	-0.00350	1.30272
……	……	……	2011/5/30	-0.02188	1.27422
2010/4/26	-0.02509	1.13768	2011/6/6	-0.02135	1.24701
2010/5/3	-0.06106	1.06822	2011/6/13	0.00482	1.25302
……	……	……	……	……	……
2010/5/17	-0.04025	1.05141	2012/2/27	0.00594	1.44347
2010/5/24	0.00227	1.05379	2012/3/5	-0.00064	1.44254
……	……	……	……	……	……
2010/7/12	-0.01016	1.03323	2012/8/6	0.01170	1.49319
2010/7/19	0.03736	1.07183	……	……	……
2010/7/26	0.00133	1.07325	2012/12/17	0.00895	1.53033
2010/8/2	0.01732	1.09184	2012/12/24	-0.02034	1.49920
2010/8/9	-0.03610	1.05243	2012/12/31	0.03841	1.55679

Source: own calculation.

It needs to be mentioned that here all the result of wealth of each week are totally based on the initial investment wealth, which is 1 dollar. From table 4.2, we can see that the wealth is 0.98322 dollars in 24/3/2008, which is wealth value for second week. but at end of in-sample period, the wealth increase to 1.55679 dollars, it also means that the most of portfolio return is positive. In this period, the minimum wealth is 0.5753 dollar, and the maximum wealth is 1.5568 dollars. In addition, we can get more information from figure 4.4, which is show that the general trend of portfolio wealth in the in-sample period. In the figure 4.4, the wealth is generally keeping the trend of increasing in these five years, of course, there exists wealth with a decline from 2008 to 2009, at that time, the American financial crisis had spread around the world, and most of the stock prices went down, these companies that we choose also has been hit by the financial crisis without exception. But after financial crisis, the wealth of portfolio was rising.

We can compute the Sharpe ratio according to the formula (3.30). Annual portfolio return is 11.96%, annual risk-free rate is 2.813%, annual standard deviation is 23.48%, so the Sharpe ratio under naive strategy is:

$$\text{Sharpe ratio} = \frac{11.96\% - 2.813\%}{23.48\%} = 0.3896. \tag{4.1}$$

In addition, there are some basic information of performance by using naive strategy are shown in table 4.3.

Table 4.3 Performance under naive strategy in in-sample period.

Final wealth	1.55679 dollars
Standard deviation(annualized)	23.48%
Annual portfolio return	11.96%
Sharpe ratio	0.3896
Weekly portfolio return	0.23%
Weekly standard deviation	3.26%
Maximum drawdown	46.22%

.

Source: own calculation.

4.2.2 Markowitz Mean-Variance model

Markowitz mean-variance model is used for analyzing the certain portfolio risk against expected portfolio return. And investors attempt to make more efficient investment choice by seeking the lowest standard deviation for a given expected return or seeking the highest expected return for a given standard deviation level. So, in Markowitz mean-variance model, we should compute the expected return and the standard deviation on each portfolio. In order to find the optimal portfolio, we need to meet some constraints. And in this part, we should obtain the efficient frontier of in-sample period.

From the figure 4.5, among the stock in this portfolio, the APPL has the highest annual expected return, GE has the lowest expected return, JNJ has the lowest level of standard deviation, and the JPM has the highest level of standard deviation.

Because of some constraints[19] have set, we can obtain these portfolios on the efficient frontier which with the higher expected return than other portfolio on a given standard deviation, or with the lower standard deviation than other portfolio on a given expected return. From the figure 4.6, the efficient frontier is showing the optimal portfolio with certain level of risk and expected return. It's easy to find that the range of annual expected return on this curve is between 5% to 60%, at the same time, the range of annual standard deviation on this curve is between 10% to 80%. then, there are ten portfolios from the efficient frontier, and their standard deviation, expected return and final wealth of each portfolio is shown in the table 4.4.

[19] Described in chapter 3.

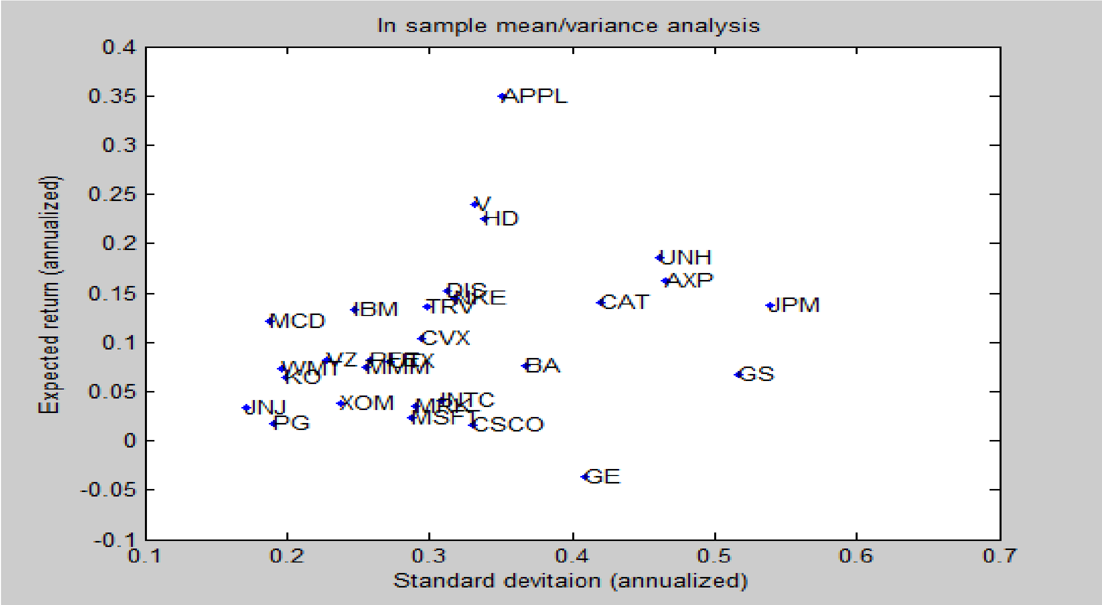


Figure 4.5 The expected return and standard deviation for each stock in the in-sample period.

Source: own calculation.

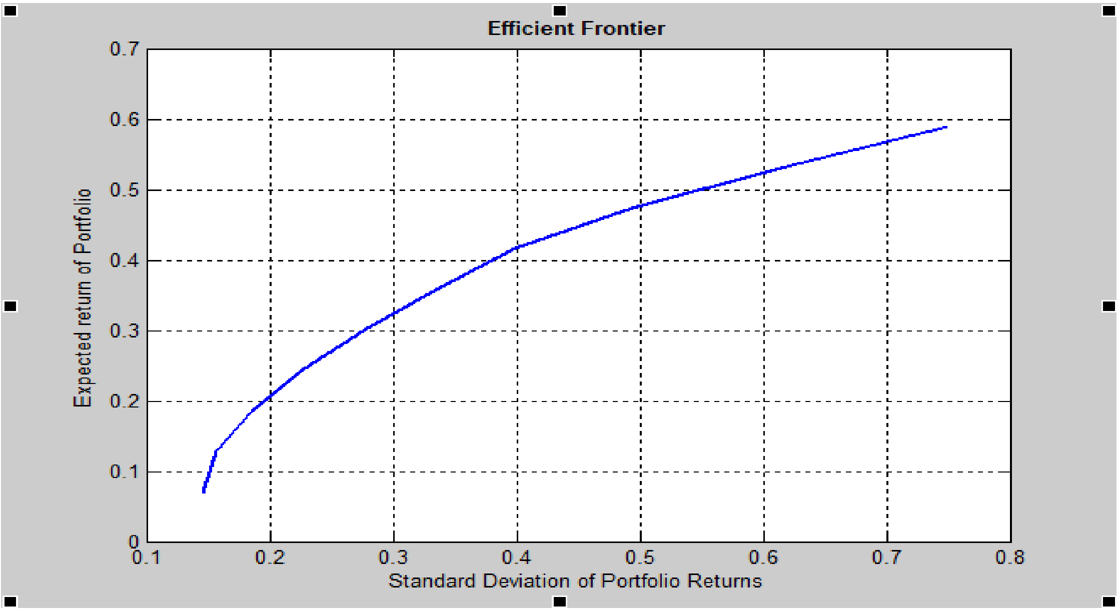


Figure 4.6 Efficient frontier in the in-sample period.

Source: own calculation.

Then, based on the standard deviation and expected return of each portfolio in table 4.4, we can compute the Sharpe ratio for each portfolio. Because the 10-year government bond rate of America equals to 2.813%, the annual risk-free rate equals to 2.813%. We can obtain the Sharpe ratio according to the formula (3.32), and the results are shown in table 4.5.

Table 4.4 The performance of each portfolio under mean-variance model in the in-sample period.

portfolio	mean return	standard deviation	final wealth
portfolio 1 with minimum standard deviation	7.0565%	14.6342%	1.332149703
portfolio 2	12.8108%	15.6925%	1.743642682
portfolio 3	18.5651%	18.5938%	2.245094794
portfolio 4	24.3194%	22.7210%	2.841063869
portfolio 5	30.0737%	27.7634%	3.523342946
portfolio 6	35.8281%	33.4854%	4.270729398
portfolio 7	41.5824%	39.8639%	5.042551513
portfolio 8	47.3367%	49.3281%	5.480443106
portfolio 9	53.0910%	61.4543%	5.268810227
portfolio 10 with maximum expected return	58.8453%	74.9617%	4.424291861

Source: own calculation.

Table 4.5 Sharpe ratio of each portfolio under mean-variance model in the in-sample period.

portfolio	sharpe ratio
portfolio 1 with minimum standard deviation	0.2899698
portfolio 2	0.6371047
portfolio 3	0.8471693
portfolio 4	0.9465447
portfolio 5	0.9818961
portfolio 6	0.9859552
portfolio 7	0.9725432
portfolio 8	0.9026033
portfolio 9	0.8181359
portfolio 10 with maximum expected return	0.7474791

Source: own calculation.

From the table 4.4 and 4.5, there are ten portfolios with different Sharpe ratio, standard deviation, expected return and final wealth. It's easy to seek that the increased in expected return is increased with standard deviation, however, in the in-sample period, the final wealth and Sharpe ratio don't follow this rule. The portfolio with maximum expected return doesn't have maximum final wealth, and its Sharpe ratio isn't the highest value. Portfolio 8 has the highest final wealth and the portfolio 6 has the highest value of Sharpe ratio. If we choose the portfolio only according to the Sharpe ratio, the portfolio 6 will be a good choice.

In addition, we can evaluate whether a performance of portfolio is stable, we can use the maximum drawdown as an indicator, it can represent the worst condition for the portfolio, the lower value, the better performance for the portfolio.

Table 4.6 The maximum drawdown for each portfolio under mean-variance model in the in-sample period.

portfolio	portfolio 1	portfolio 2	portfolio 3	portfolio 4	portfolio 5	portfolio 6	portfolio 7	portfolio 8	portfolio 9	portfolio 10
Max Drawdown	26.819%	27.127%	29.696%	38.102%	47.855%	56.810%	64.220%	72.599%	80.587%	87.589%
Mean Max Drawdown	53.140%									

Source: own calculation.

From the table 4.6, the mean of maximum drawdown with ten portfolios is53.14%, and among ten portfolios, the portfolio 1 has the lowest value of maximum drawdown, and the portfolio 10 has the highest value of maximum drawdown, it represents that there exists a relatively small loss when portfolio 1 faces the worst condition, and there exists a relatively large loss when portfolio 10 face the worst condition.

4.2.3 Max Sharpe ratio model

Max Sharpe ratio model refers to a portfolio will be set up, which the Sharpe ratio is maximized. In the in-sample period, there is a portfolio with the maximum Sharpe ratio, the weight of this portfolio is generated by using the formula (3.27). The standard deviation, expected return, Sharpe ratio, final wealth and maximum drawdown are shown in table 4.7.

Table 4.7 The performance of portfolio under max Sharpe ratio in the in-sample period.

Expected return	Standard deviation	Final wealth	Sharpe ratio	Maximum drawdown
34.0207%	31.6304%	4.113650	0.986637	54.1372%

Source: own calculation.

4.2.4 Random Model

Random model is using the *rand* function in Matlab to generate a series of random weights. Then, based on these weights to set up a series random portfolio. In order to make a better study under random model, we set up 50,000 portfolios. The histogram of portfolio mean returns, standard deviations and Sharpe ratios is shown in figure 4.3, figure 4.7 and figure 4.8.

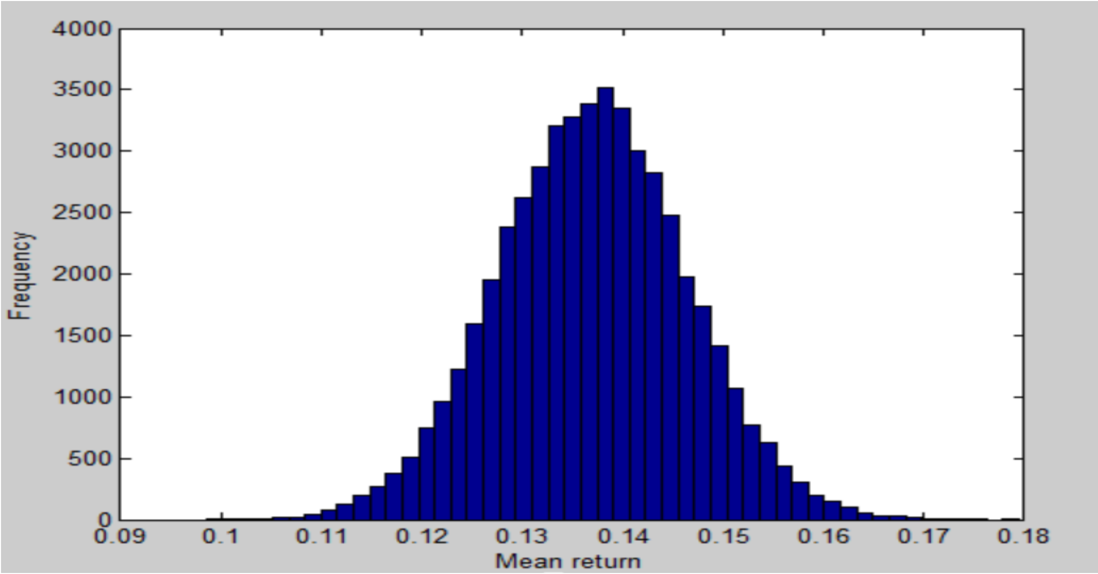


Figure 4.7 Mean return under random model in in-sample period.

Source: own calculation.

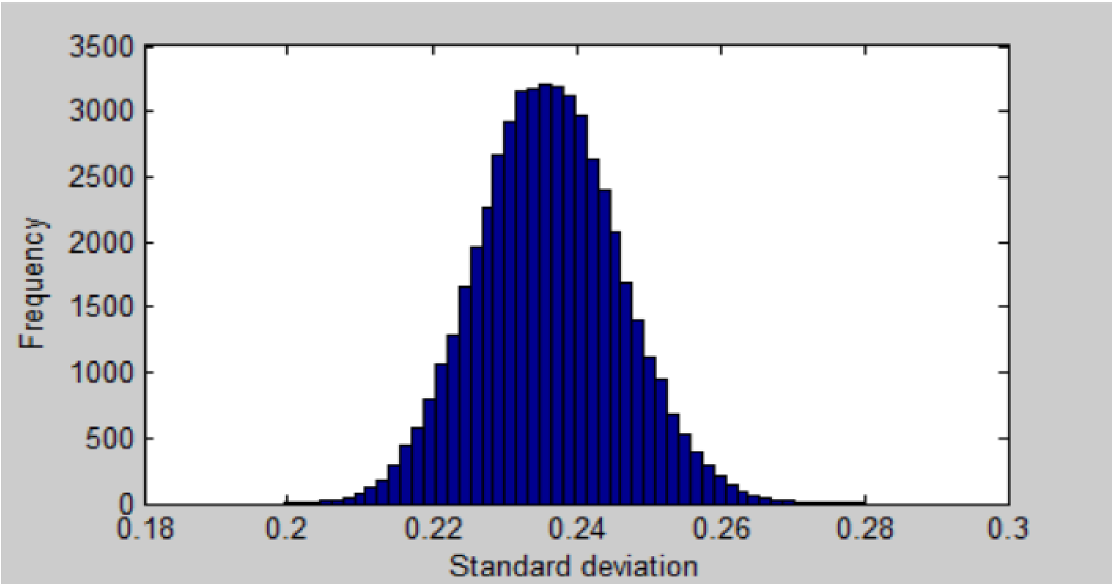


Figure 4.8 Standard deviation under random model in in-sample period.

Source: own calculation.

According to the figure 4.7, in the in-sample period, the value of mean return of portfolio is between 9% and 18%, the mean of expected return is 11.96%. The frequency at mean return mostly focused on between 10% and 14%. In addition, from the figure 4.7, the highest frequency at mean return around the value of 14% is higher than 3500. Then, after calculation in Matlab, the mode of mean return is 7.07%. The median of mean return is 11.97%.

As shown in figure 4.8, the value of standard deviation of portfolio is between 20% and 28%, the mean of standard deviation is 23.62%, and the frequency at standard deviation mostly focused on between 0.22 and 0.26, which the highest frequency at standard deviation around the value of 19.92% is higher than 3000, and the median of standard deviation is 23.6%.

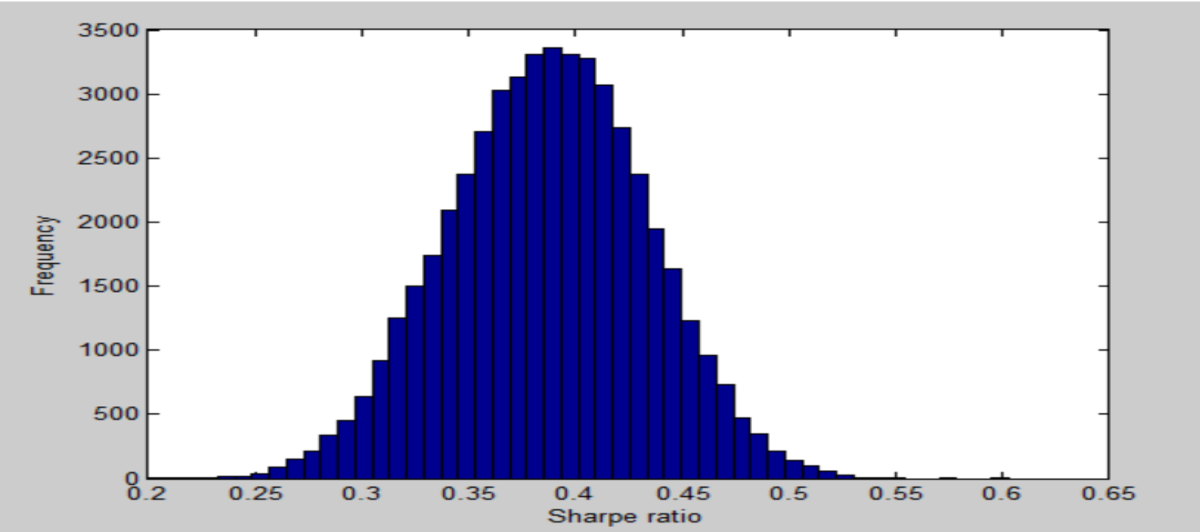


Figure 4.9 Shaper ratio under random model in in-sample period.

Source: own calculation.

In the figure 4.9, in the in-sample period, the value of Sharpe ratio of portfolio is between 0.2 to 0.6, and after calculating in Matlab, the mean of Sharpe ratio is 0.387, the mode of Sharpe ratio is 0.2003, and the highest frequency at Sharpe ratio around 0.37. The median of Sharpe ratio is 0.3877.

In the random model, there are 50,000 portfolios, it means that there are 50000 different Sharpe ratios. Now, we can make a comparison between the value of Sharpe ratio of random model and the value of mean of Sharpe ratio in other three models.

Table 4.8 Sharpe ratio comparison in the in-sample period. (Based on portfolio of random model)

	Mean-variance model	Naive strategy	Max Sharpe ratio
percentage ($Sr > S_{\text{others model}}$)	0	48.46%	0

Source: own calculation.

As shown in table 4.8, the value of percentage represents how many portfolios with Sharpe ratio in random model that are higher than other models' Sharpe ratio. So, we can see that there are not any portfolios that their Sharpe ratio are higher than the Sharpe ratio of mean-variance model and Max Sharpe ratio. However, in the random model, there are 48.46% of portfolios that its Sharpe ratio are higher than the Sharpe ratio of naive strategy. In view of the above situation, under the index of Sharpe ratio, portfolio from random model doesn’t have enough good performance in the in-sample period.

4.3 The application of models in the out-of-sample period

In previous part, the application of models of in-sample period is mentioned, we calculate the different characteristics for each portfolio under each model. And now, we will use same method to evaluate each portfolio in the out-of-sample period.

4.3.1 Naive strategy

In out-of-sample period, we have 30 different stocks, and according to the character of the naive strategy, the weight of each stock is $\frac{1}{30}$, and we can compute the return of portfolio and the wealth in each weak according to the formula (3.1) and (3.19), after that, we can obtain the out-of-sample period portfolio wealth under naive strategy, the result is shown in figure 4.10

From the figure 4.10, we can know the general trend of wealth is increasing under the strategy in the out-of-sample period, and the initial investment wealth is 1 dollar, however, at end of out-of-sample period, the wealth is near 2 dollars.

We can compute the Sharpe ratio of out-of-sample period according to the formula (3.30). Annual portfolio return is 13.7%, annual risk-free rate is 2.813%, annual standard deviation is 10.88%, so the Sharpe ratio under naive strategy is:

$$\text{Sharpe ratio} = \frac{13.7\% - 2.813\%}{10.88\%} = 1.0006.$$

Then, there are some basic information of performance by using naive strategy will be collected in table 4.9.

Figure 4.10 The trend of out-of-sample period wealth under naive strategy.

Source: own calculation.

Table 4.9 Performance under naive strategy in out-of-sample period.

Final wealth	1.9339 dollars
Annual Standard deviation	10.88%
Annual portfolio return	13.70%
Sharpe ratio	1.0006
Weekly portfolio return	0.26%
Weekly standard deviation	1.51%
Maximum drawdown	12.14%

Source: own calculation.

4.3.2 Markowitz Mean-Variance model

In this part, the Markowitz mean-variance model is applicated in the out-of-sample period. We can get the figure with the annual expected return and standard deviation on each stock, and efficient frontier of out-of-sample period is also easy to obtain.

From the figure 4.11, it's obvious that the highest expected return and the lowest expected return belong to BA and GE in this period, and the highest standard deviation and the lowest standard deviation belong to DDS and JNJ.

Then, there are ten portfolios generated according to the weights from mean-variance model in the in-sample period, and their standard deviation, expected return and final wealth of each portfolio is listed in the table 4.10.

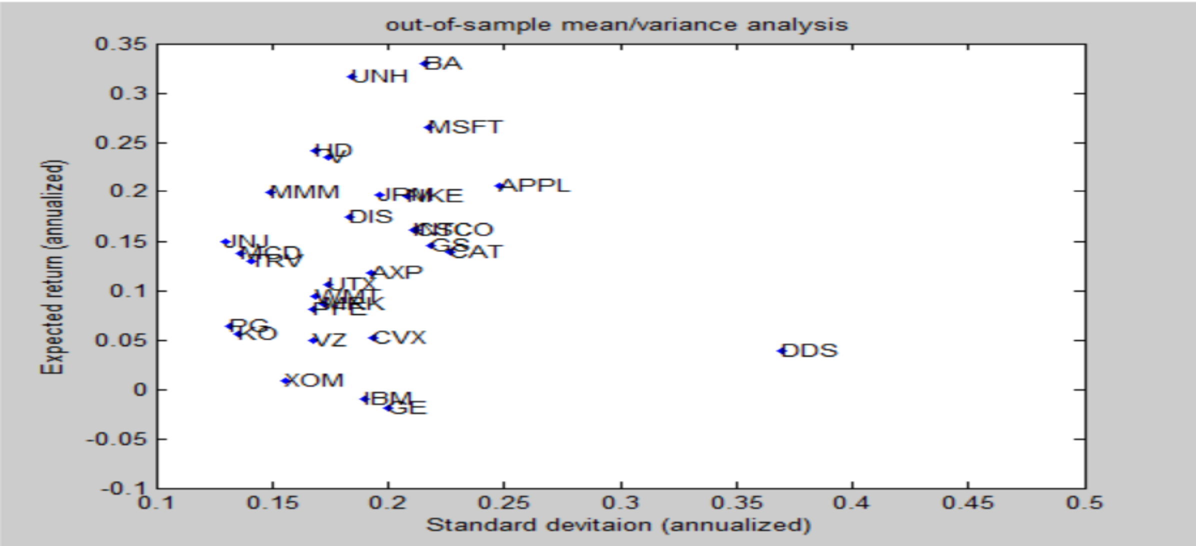


Figure 4.11 The expected return and standard deviation for each stock in the out-of-sample period.

Source: own calculation

Table 4.10 The performance of each portfolio under mean-variance model in the out-of-sample period.

portfolio	Mean return	Standard deviation	Final wealth
portfolio 1	12.1261%	9.9850%	1.79554
portfolio 2	13.8071%	10.4047%	1.94960
portfolio 3	14.8230%	11.6190%	2.03798
portfolio 4	16.0869%	13.1294%	2.15131
portfolio 5	16.8801%	15.4637%	2.20151
portfolio 6	17.2677%	18.7239%	2.18285
portfolio 7	15.9041%	21.9193%	1.97312
portfolio 8	11.9049%	24.0547%	1.57524
portfolio 9	7.9056%	29.5577%	1.19646
portfolio 10	3.9063%	36.9532%	0.86378

Source: own calculation.

From the table 4.10, to distinguish different character for each value, we use the different color to represent them: the greener color represents the better data. In the out-of-sample period, the portfolio 1 still has minimum standard deviation with 9.985%, however, the portfolio 10 doesn't have maximum expected return. In out-of-sample period, the portfolio 6 has maximum expected return with 17.2677%, the minimum expected return and maximum standard deviation belongs to portfolio 10, which are 3.9063% and 36.9532%. The maximum final wealth is obtained by portfolio 5.

After that, based on the standard deviation and expected return of each portfolio in table 4.10, we can compute the Sharpe ratio for each portfolio. The annual risk-free rate equals to 2.813%. Thus, we can obtain the Sharpe ratio according to the formula (3.32) and obtain the maximum drawdown according to formula (3.33). The results will be shown in table 4.11.

According to the description of Sharpe ratio in chapter 3, the portfolio with a higher ratio is a better choice for investing. So, from the table 4.11, the portfolio 2 will be a good choice for investment. Then, we have knew the nature of the maximum drawdown—the lower value of maximum drawdown, the better performance of portfolio. so, based on this rule, the portfolio 4 is a good choice for investment among these portfolios. After that, from the table 4.11, we can know the mean of these Sharpe ratios is 0.38742, and the mean of maximum drawdown is 29.8480%.

Table 4.11 Sharpe ratio and maximum drawdown of each portfolio under mean-variance model in the out-of-sample period.

portfolio	sharpe ratio	Maximum drawdown
portfolio 1	0.93271	15.1915%
portfolio 2	1.32700	11.9497%
portfolio 3	1.27575	11.5468%
portfolio 4	1.22526	11.4888%
portfolio 5	1.09159	16.9051%
portfolio 6	0.92223	27.6801%
portfolio 7	0.72558	37.9145%
portfolio 8	-0.13663	44.7044%
portfolio 9	0.26746	54.0977%
portfolio 10	0.10571	67.0018%
mean	0.38742	29.8480%

Source: own calculation.

4.3.3 Max Sharpe ratio model

In the out-of-sample period, we can use same method as in-sample period to obtain a portfolio with maximum Sharpe ratio. The expected return, standard deviation, final wealth, Sharpe ratio and maximum drawdown will be listed in table 4.12.

Table 4.12 Performance of portfolio under max Sharpe ratio model in out-of-sample period.

Mean return	Standard deviation	Wealth	Sharpe ratio	Maximum drawdown
17.1460%	17.6391%	2.19113	0.81257	24.277%

Source: own calculation.

4.3.4 Random Model

As in-sample period, we set up 50000 random portfolios based on the random weights, which are generated by *Rand* function of Matlab. And the histogram of portfolio expected return, standard deviation and Sharpe ratio are shown in figure 4.12, figure 4.13 and 4.14.

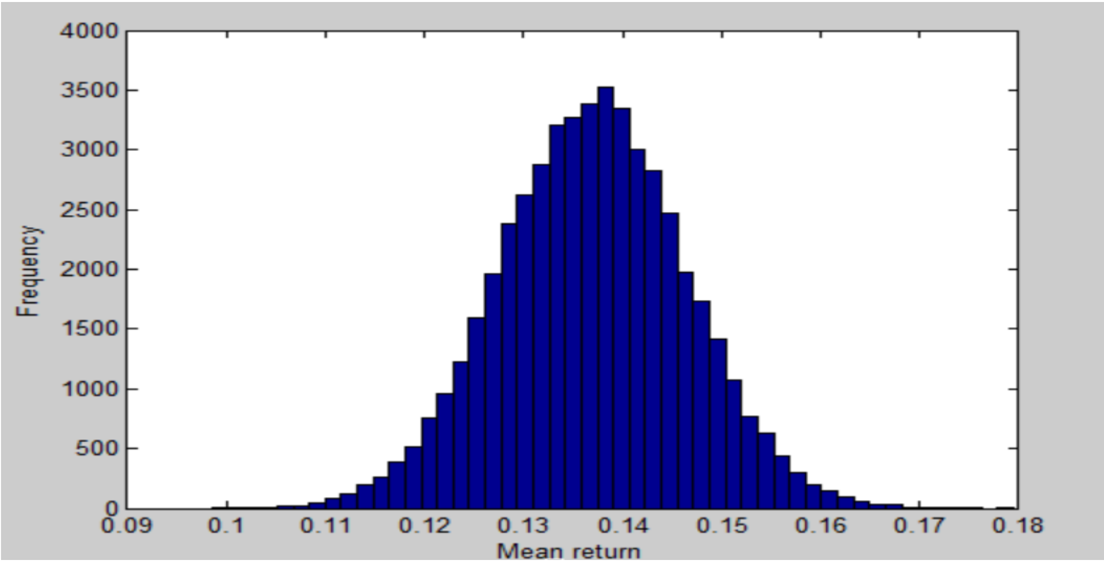


Figure 4.12 Mean return under random model in the out-of-sample period. (Annualized)
Source: own calculation.

According to the figure 4.12 in the out-of-sample period, the value of mean return of portfolio is between 9% and 18%, And the highest frequency at expected return with 0.135 is higher than 3500.The frequency at expected return mostly focused on between 0.12 and 0.15. After computing in Matlab, the mode of mean return is 9.15%, median of mean return is 13.7%, the mean of mean return is 13.7% as well.

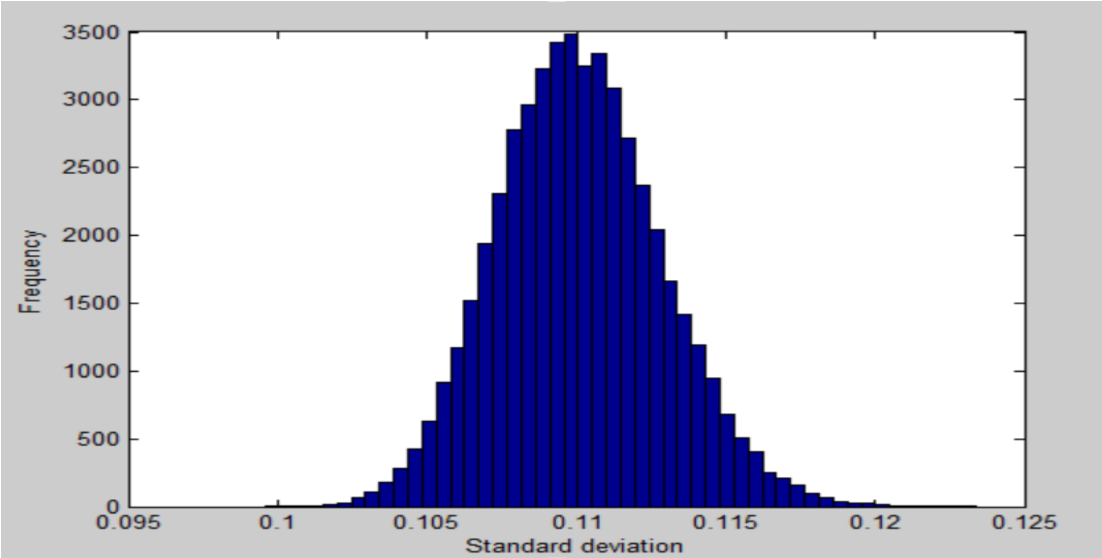


Figure 4.13 Standard deviation under random model in the out-of-sample period. (Annualized)
Source: own calculation.

And in figure 4.13, the value of standard deviation of portfolio is between 0.95% and 12.5%, and the frequency at standard deviation mostly focused on between 0.105 and 0.115, which the highest frequency at standard deviation with 0.11 is near 3500. After computing in Matlab, the mean of standard deviation is 11.01%, the mode of standard deviation is 9.96%, and the median of standard deviation is 11%.

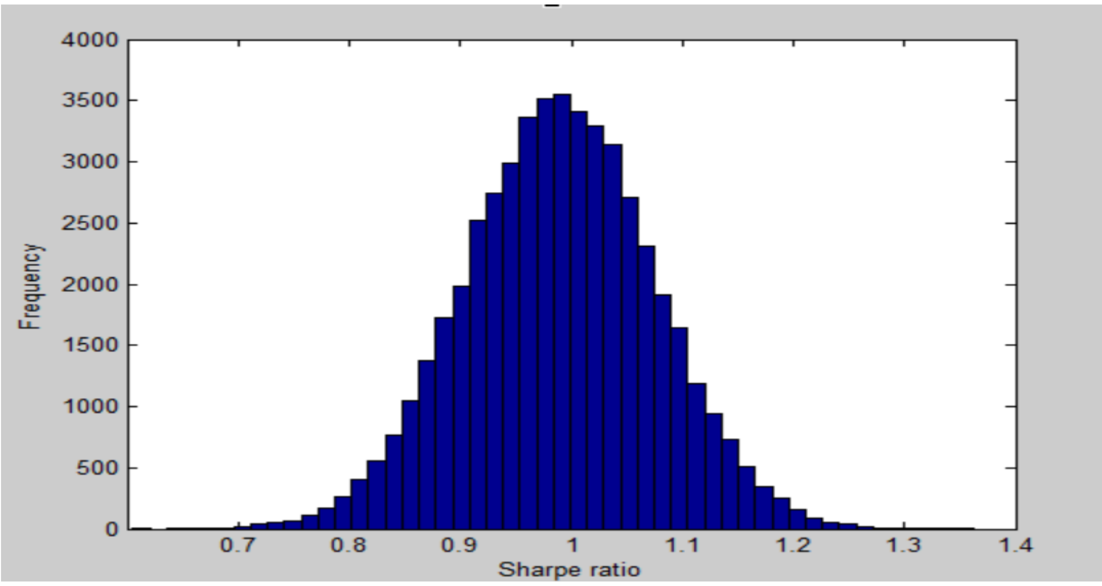


Figure 4.14 Sharpe ratio under random model in the out-of-sample period. (annualized)
Source: own calculation.

As shown in figure 4.14, in the out-of-sample period, the value of Sharpe ratio of portfolio is between 0.7 to 1.4, and it's easy to see that the highest frequency at Sharpe ratio around the value of 1. After calculation, the mean of Sharpe ratio is 0.9891, the mode of Sharpe ratio is 0.604, and the median of Sharpe ratio is 0.9891.

Table 4.13 Sharpe ratio comparison in the out-of-sample period. (Based on portfolio of random model)

	Mean-variance model	Naive strategy	Max Sharpe ratio
percentage ($Sr>S_{others\ model}$)	100%	44.83%	97.89%

Source: own calculation.

As shown in table 4.13, the value of percentage represents how many portfolios with Sharpe ratio in random model that are higher than other models' Sharpe ratio. So, we can see that all portfolios that their Sharpe ratios are higher than the Sharpe ratio of mean-variance model. And in the random model, there are 44.83% of portfolios that its Sharpe ratio are higher than the Sharpe ratio of naive strategy. Meanwhile, there are 97.89% of portfolio that its Sharpe ratio are higher than the Sharpe ratio of max Sharpe ratio. In a word, in the out-of-sample period, the value of Sharpe ratio indicates that there is a good financial performance for these portfolios under random model.

4.4 Comparison of different periods and different models

In previous parts, we divide the whole sample period into in-sample period (17/3/2008-31/12/2012) and out-of-sample period (7/1/2013-15/1/2018). Based on four different models, the expected return, standard deviation, wealth, Sharpe ratio are calculated in the different periods, respectively. In this part, we make a comparison of performance under the different models in the different period.

In the figure 4.15, it includes two scatter plots, which the left plot shows the expected return and standard deviation in the in-sample period, and the right plot shows the expected return and standard deviation in the out-of-sample period. Compared with in-sample period, the value of standard deviation of all portfolios is lower in out-of-sample period, of course, the expected return is lower than that of in-sample period as well, for example, in the in-sample period, the APPL has the value of expected return with more than 35%, but in out-of-sample period, the value of expected return is lower than 25%, and the GE has the expected return with -5% and the standard deviation with near 40% in the in-sample period, but in out-of-sample period, its expected return is increased, but it's till negative, and the standard deviation is decreased, which is near 20%, it means that the risk level of this stock is declining in the out-of-sample period.

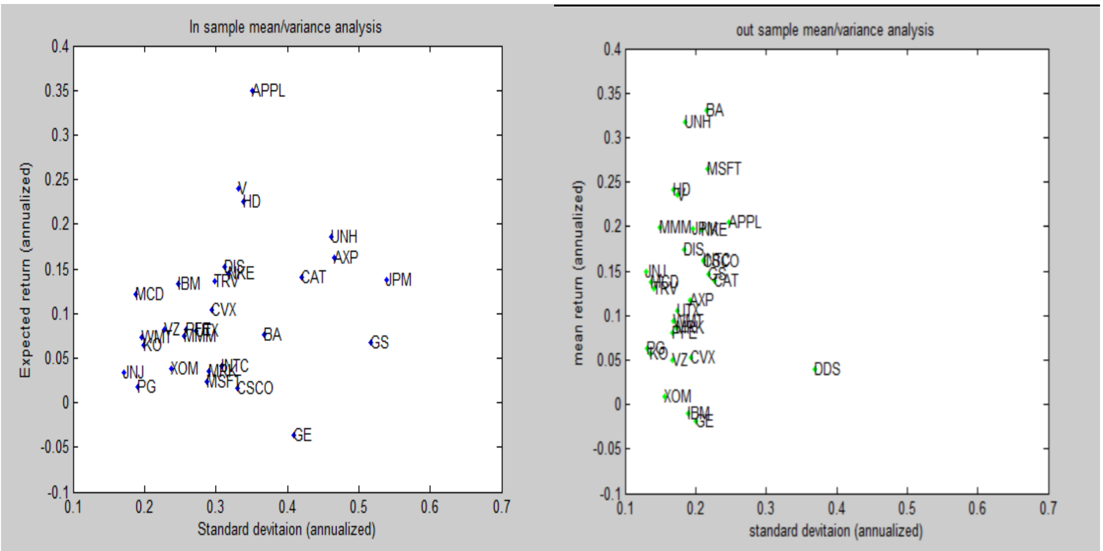


Figure 4.15 The standard deviation and expected return on each stock in different periods. source: own calculation.

The figure 4.16 shows the portfolios set up by using four different models in two different periods. The left scatter plot represents the portfolio in the in-sample period, and the right scatter plot represents the portfolio in the out-of-sample period. The red point indicates that one portfolio is set up under the max Sharpe ratio model, ten blue points indicate that ten portfolios are set up under the mean-variance model, the green point refers to one portfolio that is set up under the naive strategy, and the last one is random model, which is shown out as 50000 points with gray, and it represents that there are 50,000 portfolios.

Under the max Sharpe ratio, the standard deviation of portfolio falls from about 30% to about 17%, of course, the expected return of portfolio falls down as well, from about 35% to about 20%, it indicates that it isn't a good result when the weight of in-sample period uses in out-of-sample period.

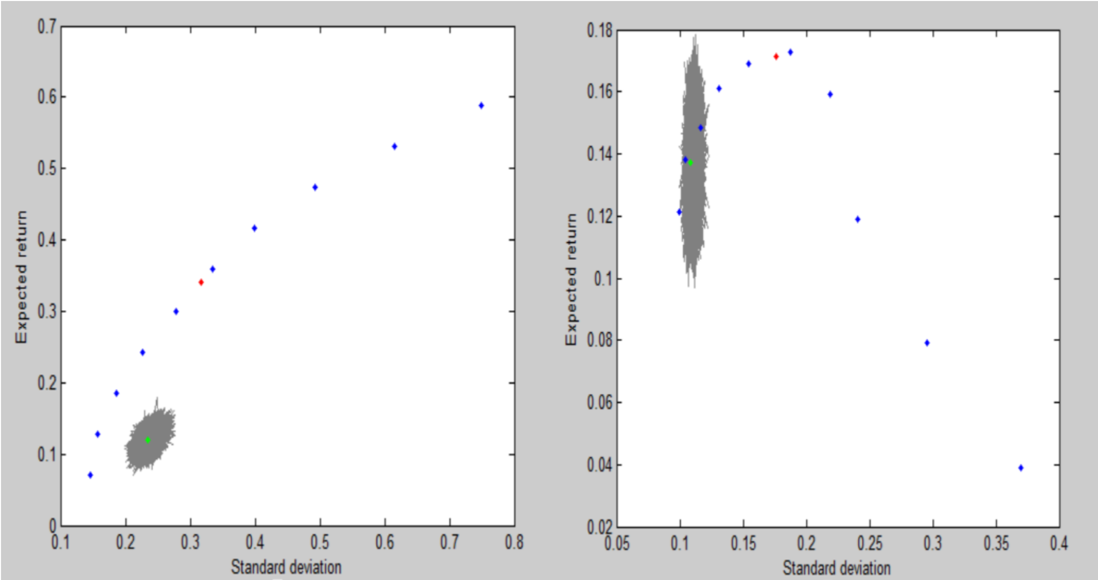


Figure 4.16 The standard deviation and expected return of portfolio under different models in different periods.

Source: own calculation.

Then, under the naive strategy, the standard deviation of portfolio falls from about 20% to about 11%, however, the expected return hasn't change much, it means that this portfolio with the weight of in-sample period will obtain a certain return at a lower risk.

Next, under the random model, there are 50000 portfolio, we can see that the range of standard deviation is from 20% to 30% in the in-sample period, and the range of expected return is from 8% to 15% in the in-sample period, but in out-of-sample period, the range of standard deviation is from 10% to 12%, and the range of the expected return is from 10% to 18%, it represents that these portfolios with the weight of in-sample period will have a good return and a lower risk in the out-of-sample period.

Finally, under the mean-variance model, there are ten portfolios. Compared with in-sample period, it's easy to find these points with a lower standard deviation and a lower expected return in out-of-sample period in addition to the first portfolio. The first portfolio of mean-variance model has the higher return and the lower risk compared with the first portfolio in the in-sample period. But from the seventh portfolio to the tenth portfolio, the standard deviation is increasing, and the expected return is decreasing. It's not a good sign for the investors if they invest in these portfolios with the weight of mean-variance model.

In addition to expected return and standard deviation, Sharpe ratio also can be a good index to measure a portfolio.

Table 4.14 Sharpe ratio under the different model in different period.

portfolio	in-sample period	out-of-sample period
max Sharpe ratio	0.9866	0.8126
mean-variance model (minimum value)	0.2900	0.0296
mean-variance model (mean value)	0.8354	0.3874
mean-variance model (maximum value)	0.9860	1.0566
random model (minimum value)	0.2124	0.6040
random model (mean value)	0.3879	0.9884
random model (maximum value)	0.5875	1.3466
naive strategy	0.3895	1.0006

Source: own calculation.

The table 4.14 shows the result of Sharpe ratio under these four models in different period. In the in-sample period, the value of Sharpe ratio under the max Sharpe ratio is the highest, followed by mean-variance model, naive strategy and random model. However, in the out-of-sample period, the highest ratio is under naive strategy, followed by random model, max Sharpe ratio model and mean-variance model. What's more, under the max Sharpe ratio model, the Sharpe ratio of out-of-sample period is lower than the ratio in the in-sample period, and the similar things take place in the mean-variance model, but the maximum value of Sharpe ratio in out-of-sample period is higher than the value in in-sample period. Comprehensively, it won't be a better result under these two models when the weight of in-sample period uses in out-of-sample period. However, under the random model and naive strategy model, Whichever kind of value you look at it, the Sharpe ratio of out-of-sample period is higher than the ratio in the in-sample period, it means that will be a better result when the weight of in-sample period uses in out-of-sample period.

As an indicator of downside risk over specified time period, maximum drawdown has high reference value for investment.

The table 4.15 shows the result of max drawdown under these four models in different period. Among these four models, the maximum drawdown of naive strategy has the largest falls from in-sample period to out-of-sample period, but overall, the values of maximum drawdown under four models were decreased greatly, which reflects the reduction of loss under different models in the out-of-sample period, this is a good sign for investors.

Table 4.15 Maximum drawdown under the different model in different period.

	In-sample period	out-of-sample period
Naive strategy	46.22%	12.14%
Mean-variance model (minimum value)	26.82%	11.49%
Mean-variance model (mean value)	53.14%	29.85%
Mean-variance model (maximum value)	87.59%	67.00%
Max Sharpe ratio	54.14%	24.28%
Random model (minimum value)	26.82%	11.49%
Random model (mean value)	46.12%	12.47%
Random model (maximum value)	87.59%	67.00%

Source: own calculation.

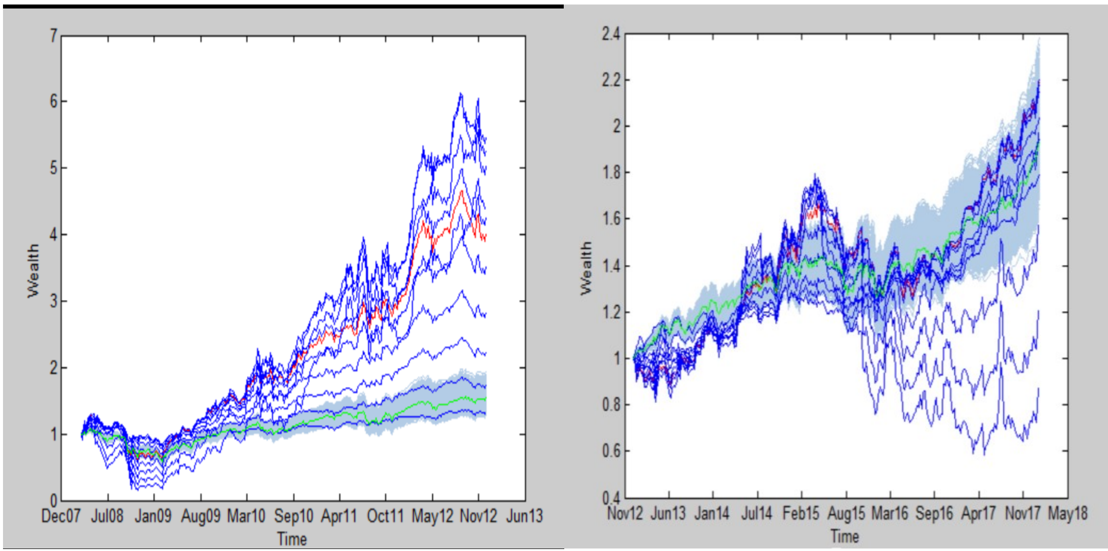


Figure 4.17 The trend of wealth under different model in different period.

Source: own calculation.

We assume that the initial investment is 1 dollar for all portfolios. In the figure 4.17, the left figure is wealth of in-sample period, and the right figure is wealth of out-of-sample period. the red curve represents the wealth under max Sharpe ratio, green curve represents the wealth under naive strategy, the blue curve represents the wealth under mean-variance model and the grey curve is the wealth under the random model. Here we have mentioned that there was financial crisis in 2008, so, we can see the wealth curves have a decline in 2008, but after this year, in in-sample period, all portfolios are keeping the trend of increasing. Among these four models, the wealth under naive strategy and random model doesn't have a large increase in in-sample period, which the amount of increase is smaller than 1 dollar. But under the rest of models, the amount of increase is larger. Under the max Sharpe ratio, the wealth of portfolio increases from 1 dollar to more than 4 dollars. And under the mean-variance model, here exists that several portfolios have a large increase in in-sample period, but it still has some portfolios don't have a large increase. The range of increasing is from lower than 1 dollar to more than 5 dollars. However, I need to mention that the wealth exists volatility from 2011 to 2012. Compared with in-sample period, the wealth curve is more complicated in the out-of-sample period. Among these four models, the portfolios with random model have a big increase, and the general trend of wealth is increasing in this period, which the highest value of wealth reach 2.4 dollars. Next, the portfolio with naive strategy also has a good trend, but there is a slow growth from 2015 to 2016. By comparison, the wealth trend of portfolios with max Sharpe ratio model and mean-variance model is not smooth in the out-of-sample period. Under the mean-variance model, the most of portfolio kept the trend of increasing from 2012 to 2015, which the value of highest wealth reach 1.8 dollars. But since 2015 the wealth has been decreasing, the wealth of some portfolios even lower than 1 dollars during recent 2 years, of course, there also exist the wealth path of some portfolio began to fall down in 2015, and rise in 2016, even the wealth of some portfolios of mean-variance reach 2.2 dollars. Then, the wealth trend of portfolio with max Sharpe ratio model is similar with mean-variance model, which the value of wealth moves up and down in this period. So, among four models, the portfolios from random model and naive strategy will have a good performance in the out-of-sample period.

The above results are all calculated by applying different models, and now, there is a chart from Yahoo, which shows the price trend of Dow Jones Industrial Average(DJIA). Because the chosen stocks are also from the DJIA, we can make a comparison between the wealth trend we calculate and the price trend of DJIA. And the performance of DJIA is also shown in table 4.16

Figure 4.18 The price trend of Dow Jones Industrial Average. (Currency in USD)

Source: https://finance.yahoo.com.

Table 4.16 The performance of DJIA. (Annualized)

Mean return	8.932%
Standard deviation	16.951%
Sharpe ratio	0.36097
Maximum drawdown	49.251%

Source: own calculation.

In the figure 4.18, there was a big decrease from 2008 to 2009, and this trend is similar with wealth trend (figure 4.17) from 2008 to 2009. And over next few years, the wealth trend is much the same with random model and naive strategy from the figure 4.17, the general trend is all keeping the increasing, and the curves both have some small decreases from 2011 to 2012 and from 2015 to 2016. We can find that the wealth trend under random model and naive strategy we estimated is roughly equivalent to the official price trend of DJIA.

5 Conclusion

In this thesis, we apply the portfolio optimization theories to seek the optimal portfolio with lower risk and higher return. There exist four models that are applied to generate different portfolios, they are the naive strategy, mean-variance model, Max Sharpe ratio model, and random model. We need to evaluate the performance of each portfolio under different model in different sample period so that choose correct model to invest.

Under the condition of applying the naive strategy, we generate one portfolio that each stock in this portfolio has the same weight. No matter from the result of Sharpe ratio, maximum drawdown or mean return, the performance of portfolio under naive strategy in the out-of-sample period is better than the performance in the in-sample period.

Under the condition of applying the mean-variance model, we generate ten different portfolios. In the in-sample period, these portfolios are efficient portfolios, however, in the out-of-sample period, some performances of these portfolios are worse than performance in the in-sample period. For Instance, the Sharpe ratio of out-of-sample period is lower than Sharpe ratio of in-sample period. Although the value of maximum drawdown is lower than that value in the in-sample period, the wealth of some of portfolio in the late out-of-sample period are even lower than 1 dollar.

Then, under the condition of applying the max Sharpe ratio model, there is one portfolio with maximum Sharpe ratio that is generated. In the in-sample period, this portfolio has the maximum Sharpe ratio, the relatively higher return and lower risk, and the wealth with increasing trend, however, in the out-of-sample period, the value of Sharpe ratio is lower than before, at the same time, the return is decreasing as well, in addition, the wealth trend is not stable, but we can ignore that the value of maximum drawdown decreased in this period.

Finally, under the condition of applying random model, there are 50,000 different portfolios that are generated. According to the description in previous chapter, the performance of portfolios of random model is stable. We all know that the ratio of increasing in mean return is not significant, but, we can see when the mean return of portfolio of other models is showing the decreased trend in the out-of-sample period, the mean return of portfolio of random model is stable and it even has a slight increase. In addition, from the results of Sharpe ratio and maximum drawdown, it should be clear that the portfolios of random model have better performance compare with other models.

Of course, if we only compare the results of performance of out-of-sample period, the portfolios in random model still are good choice for investment. The reason is that these portfolios all have the better results in all kinds of performance indicators, such as the highest Sharpe ratio, the comparably low maximum drawdown, stable return and the lower risk.

From the results of this thesis, no matter which aspect of performance we consider, the portfolio of random model is a relatively good and safe option for investors.

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[12]http://s2.q4cdn.com/661678649/files/doc_financials/annual/2017/2017-Annual-Report.pdf

[13] https://cn.investing.com/rates-bonds/u.s.-10-year-bond-yield

List of Abbreviations

$E(R_p)$	The expected return of portfolio
Q	Covariance matrix
σ_p^2	Variance of the portfolio
S_k	Skewness
CVaR	Conditional Value at Risk
VaR	Value at Risk
W_t	The wealth of initial investment
R_f	The risk-free rate
DD_t	Drawdown
$MDD_{0,T}$	Maximum drawdown
$\rho(X)$	The mapping function from random variable to real numbers

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- I am informed that Act No. 121/2000 Coll. – the Copyright Act, in particular, Section 35 – Utilisation of the Work as a Part of Civil and Religious Ceremonies, as a Part of School Performances and the Utilisation of a School Work – and Section 60 – School Work, fully applies to my diploma (bachelor) thesis;
- I take account of the VSB – Technical University of Ostrava (hereinafter as VSB-TUO) having the right to utilize the diploma (bachelor) thesis (under Section 35(3)) unprofitably and for own use ;
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Ostrava dated 27. 4. 2018

SHUFAN CHEN

....

Student's name and surname

List of Annexes

Annex A Matlab Programs

Program A1 Data Characteristics

```
% characteristics of the returns of the assets (in sample period)
T=readtable('stock1.xlsx')
symbol=T.Properties.VariableNames(2:end)'
weeklyreturn=T{2:end,2 : end}./T{1:end-1,2 : end}-1
m=mean(weeklyreturn)
amean=m*52
s=std(weeklyreturn)
astd=s*sqrt(52)
med=median(weeklyreturn)
amed=med*52
ske= skewness(weeklyreturn)
kur=kurtosis(weeklyreturn)
figure;
plot(astd,amean,'.');
xlabel('Standard devitaion (annualized)');
ylabel('Expected return (annualized)');
title('In sample mean/variance analysis');
axis([0.1 0.7 -0.1 0.4]);
% legend put
% legend(symbol) %% put the legend, the best would be in graph
text(astd,amean,symbol)
% characteristics of the returns of the assets (out sample period)
T2=readtable('stock2.xlsx')
symbol2=T2.Properties.VariableNames(2:end)'
weeklyreturn2 =T2{2:end,2 : end}./T2{1:end-1,2 : end}-1
m2=mean(weeklyreturn2)
amean2=m2*52
s2=std(weeklyreturn2)
astd2=s2*sqrt(52)
med2=median(weeklyreturn2)
amed2=med2*52
ske2= skewness(weeklyreturn2)
kur2=kurtosis(weeklyreturn2)
%figure of portfolio in the in-sample period.
plot(astd,amean,'.g');
xlabel('Standard devitaion (annualized)');
ylabel('Expected return (annualized)');
```

```
title('In sample mean/variance analysis');
axis([0.1 0.7 -0.1 0.4]);
text(astd,amean,symbol);
```

Program A2 Generate the weight under different model

```
%% Estimate the weights
p=Portfolio('stock1',symbol2,'RiskFreeRate',0.02813/52) % look what
is the T-bill rate and chose it according to it(2.813%)
p=estimateAssetMoments(p,weeklyreturn)
p=setDefaultConstraints(p)
ws=estimateMaxSharpeRatio(p)% weight under max Sharpe ratio
me=mean(weeklyreturn)
covariance=cov(weeklyreturn)*52
variance=var(weeklyreturn2)*52
p = Portfolio
p = setAssetMoments(p, amean, covariance)
p = setDefaultConstraints(p)
[risk,ret]=plotFrontier(p) %Efficient frontier figure
we = estimateFrontier(p) %weight under mean-variance model

save results;
%% Genarate random weights
wr=rand(30,50000);
for i=1:size(wr,2)
   wr(:,i)=wr(:,i)./sum(wr(:,i));
end
%% naive
naweight=repmat(1/30,30,1);
nareturn=weeklyreturn2*naweight;
%%wealth
nawea=[1;cumprod(1+nareturn)];

%% put all the weights together
w=[ws we wr naweight];
```

Program A3 The performance of each model in the in-sample period

```
%% compute the in-sample period returns,wealth paths, Sharpe
ratio, maximum drawdown

returnsoof=weeklyreturn*w; %we can change the weeklyreturn for
(out) or( in)
% rows are week
% colums are strategies 1 - maximizing sharpe 2-11 from
efficient frontier
% 12-50011 is random
% 50012 is naive
mretoos=mean(returnsoof)*52;
stdretoos=std(returnsoof)*sqrt(52);
% VaR
figure;
hist(mretoos(1,12:50011),50)

figure;
hist(stdretoos(1,12:50011),50)
% sharpe ratio
figure;
riskfree=0.02813;
sharpe1=(mretoos(12:50011)-
riskfree)./stdretoos(12:50011); %random model
hist(sharpe1,50)
sharpe3=(mretoos(2:11)-riskfree)/stdretoos(2:11); %mean-variance
model
sharpe33=(mretoos(2:11)-riskfree)./stdretoos(2:11);
shmvmax=max(sharpe33); % the max value in mean variance model
shrandmax=max(sharpe1); % the max value in random model
shrandmin=min(sharpe1); % the min value in random model
shmvdmin=min(sharpe33);% the min value in mean variance model
sharpe2=(mretoos(1:1)-riskfree)/stdretoos(1:1);% max Sharpe
ratio model
sharpe4=(mretoos(50012:50012)-riskfree)/stdretoos(50012:50012);%
naive strategy
ss2=sum(sum(sharpe1>sharpe3)); %compare with mean variance
ss3=sum(sum(sharpe1>sharpe4)); %compare with naive
ss1=sum(sum(sharpe1>sharpe2)); % compare with max sharpe ratio
```

```
figure;
plot(stdretoos(1,12:50011),mretoos(1,12:50011),'color',[0.5 0.5 
0.5]);
hold on;
plot(stdretoos(1,1),mretoos(1,1),'.r');
plot(stdretoos(1,2:11),mretoos(1,2:11),'.b');
plot(stdretoos(1,50012),mretoos(1,50012),'.g');

%compute wealth paths

wealth=cumprod(1+returnsoof);
wealth=[ones(1,size(wealth,2)); wealth];

% max drawdown
MaxDD=maxdrawdown(wealth);
mMDD1=mean(MaxDD(2:11)); % mean-variance mean result
mMDD2=mean(MaxDD(12:50011)); % random model mean result
%%
figure;
plot(stock1,wealth(:,12:50011),'color',[0.7 0.8 0.9]);
dateaxis('x',10);;
hold on;
plot(stock1,wealth(:,1),'r');
plot(stock1,wealth(:,2:11),'b');
plot(stock1,wealth(:,50012),'g')
```

Program A4 The performance of each model in the out-of-sample period

```
%% compute the in-sample period returns,wealth paths, Sharpe
ratio, maximum drawdown

returnsoof=weeklyreturn2*w; %we can change the weeklyreturn for
(out) or( in)
% rows are week
% colums are strategies 1 - maximizing sharpe 2-11 from
efficient frontier
% 12-50011 is random
% 50012 is naive
mretoos=mean(returnsoof)*52;
stdretoos=std(returnsoof)*sqrt(52);
% VaR
figure;
hist(mretoos(1,12:50011),50)

figure;
hist(stdretoos(1,12:50011),50)
% sharpe ratio
figure;
riskfree=0.02813;
sharpe1=(mretoos(12:50011)-
riskfree)./stdretoos(12:50011); %random model
hist(sharpe1,50)
sharpe3=(mretoos(2:11)-riskfree)/stdretoos(2:11); %mean-variance
model
sharpe33=(mretoos(2:11)-riskfree)./stdretoos(2:11);
shmvmax=max(sharpe33); % the max value in mean variance model
shrandmax=max(sharpe1); % the max value in random model
shrandmin=min(sharpe1); % the min value in random model
shmvdmin=min(sharpe33);% the min value in mean variance model
sharpe2=(mretoos(1:1)-riskfree)/stdretoos(1:1);% max Sharpe
ratio model
sharpe4=(mretoos(50012:50012)-riskfree)/stdretoos(50012:50012);%
naive strategy
ss2=sum(sum(sharpe1>sharpe3)); %compare with mean variance
ss3=sum(sum(sharpe1>sharpe4)); %compare with naive
ss1=sum(sum(sharpe1>sharpe2)); % compare with max sharpe ratio
```

```
figure;
plot(stdretoos(1,12:50011),mretoos(1,12:50011),'color',[0.5 0.5
0.5]);
hold on;
plot(stdretoos(1,1),mretoos(1,1),'.r');
plot(stdretoos(1,2:11),mretoos(1,2:11),'.b');
plot(stdretoos(1,50012),mretoos(1,50012),'.g');

%compute wealth paths

wealth=cumprod(1+returnsoof);
wealth=[ones(1,size(wealth,2)); wealth];

% max drawdown
MaxDD=maxdrawdown(wealth);
mMDD1=mean(MaxDD(2:11)); % mean-variance mean result
mMDD2=mean(MaxDD(12:50011)); % random model mean result
%%
figure;
plot(stock2,wealth(:,12:50011),'color',[0.7 0.8 0.9]);
dateaxis('x',10);;
hold on;
plot(stock2,wealth(:,1),'r');
plot(stock2,wealth(:,2:11),'b');
plot(stock2,wealth(:,50012),'g')
```

Program A5 The performance of DJIA

```
%% DIJ weeklyreturn
T1=readtable('DDDD.xlsx');
weeklyreturnD=T1{2:end,2 : end}./T1{1:end-1,2 : end}-1;
mD=mean(weeklyreturn);
ameanD=mD*52;
sD=std(weeklyreturn);
astD=sD*sqrt(52);
riskfree=0.02813;
sharpeD=(ameanD-riskfree)./astD;
wealthD=cumprod(1+weeklyreturn);
wealthD=[ones(1,size(wealthD,2)); wealthD];
MaxDD22=maxdrawdown(wealthD);
```